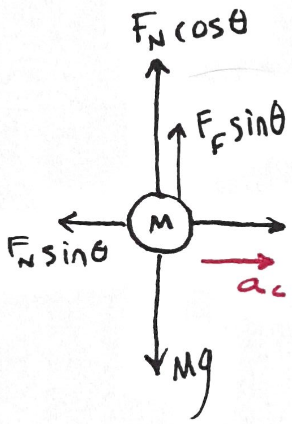
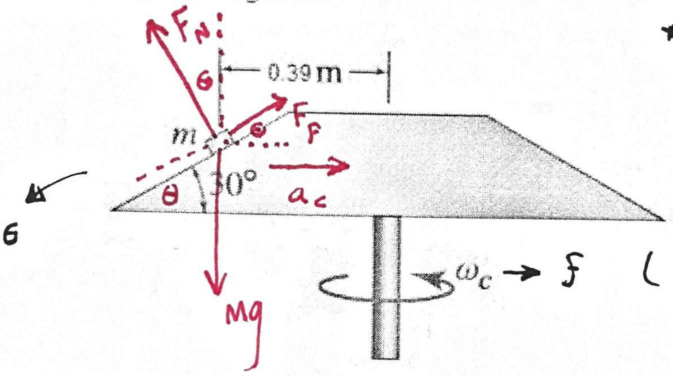
Block on Rotating Conical Surface

Name: SOLUTION Date: ____________

The small object of mass m is placed on the rotating conical surface at the radius shown. If the coefficient of static friction between the object and the rotating surface is 0.6, calculate the maximum angular velocity w_c of the cone about the vertical axis for which the object will not slip. Assume the w_c is increased very gradually so that the angular acceleration of the cone can be ignored.





* break forces into vertical and radial components

$\omega_c \rightarrow f$ (we are solving for f and can convert to angular velocity after)

[For this problem, replace the angular velocity with the frequency of rotation, *f*.]

$$F_f = \mu F_N$$

vertical

$$\Sigma F = Ma = 0$$

$$F_N\cos\theta + F_F\sin\theta - Mg = 0$$

$$F_N\cos\theta + \mu F_N\sin\theta - Mg = 0$$

$$F_N(\cos\theta + \mu\sin\theta) = Mg$$

$$F_N = \frac{mg}{(\cos\theta + \mu\sin\theta)}$$

radial

$$\Sigma F = Ma_c$$

$$F_f\cos\theta - F_N\sin\theta = m4\pi^2 r f^2$$

$$\mu F_N\cos\theta - F_N\sin\theta = m4\pi^2 r f^2$$

$$F_N(\mu\cos\theta - \sin\theta) = m4\pi^2 r f^2$$

$$\frac{mg(\mu\cos\theta - \sin\theta)}{(\cos\theta + \mu\sin\theta)} = m4\pi^2 r f^2$$

$$f = \sqrt{\frac{g(\mu\cos\theta - \sin\theta)}{4\pi^2 r(\cos\theta + \mu\sin\theta)}}$$

$$f = 0.104\text{ Hz}$$ ✓

$$\omega = 6.28 \times f = 0.65\text{ rad/s}$$