

# DYNAMICS

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The problems have been sub-divided into three groups for each mid-term (A and B) and the final is comprehensive. Problems with ♦ and ♦♦ are considered moderate to difficult and those with ♦♦♦ are perhaps the most difficult.

Successful problem solving involves the following steps:

- 1) read the question and retrieve all relevant information
- 2) draw sketches and free body diagrams
- 3) identify the governing equation
- 4) mathematically solve the problem
- 5) find the answer(s) and double-check when possible

## PROBLEMS A

### Introduction

1. The weight of one dozen apples is 5 lb. Determine the average weight and mass of one apple in both SI and U.S. units. Is it true that an average apple weighs 1 N?  
*Ans.*  $m = 0.1888 \text{ kg}$ ,  $m = 0.01294 \text{ slugs}$ ,  $W = 1.853 \text{ N}$
  
2. A pressure transducer measures a value of 200 lb/in<sup>2</sup>. Determine the value of the pressure in pascals. A pascal (Pa) is one Newton per meter squared.
  
3. In the equation  $T = \frac{1}{2} I \omega^2$ , the term  $I$  is the mass moment of inertia in kg-m<sup>2</sup> and  $\omega$  is the angular velocity in s<sup>-1</sup>.
  - (a) What are the SI units of  $T$ ?
  - (b) If the value of  $T$  is 200 when  $I$  is in kg-m<sup>2</sup> and  $\omega$  is in s<sup>-1</sup>, what is the value of  $T$  when it is expressed in terms of U.S. Customary base units?

# KINEMATICS OF PARTICLES

## Rectilinear Motion

4. The acceleration of a point in  $\text{m/s}^2$  is  $a = 20t$ . When  $t = 0$ ,  $s = 40$  m and  $v = -10$  m/s.  
What are the position and velocity of the object at  $t = 3$  s?

5. ♦ The velocity of an object in  $\text{m/s}$  is  $v = 200 - 2t^2$ . When  $t = 3$  s, its position is  $s = 600$  m.  
Determine the position and acceleration of the object at  $t = 5$  s?

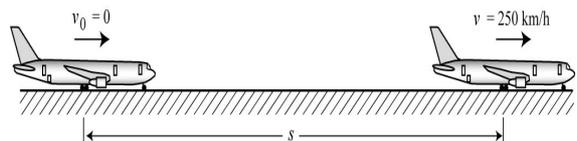
6. ♦ A projectile is fired vertically with an initial velocity of 200 m/s. Calculate the maximum altitude  $h$  reached by the projectile and the time  $t$  after firing for it to return to the ground. Neglect air resistance and assume a constant gravitational acceleration at  $9.81 \text{ m/s}^2$ .

*Ans.*  $h = 2040$  m,  $t = 40.8$  s

7. ♦♦ The acceleration of an object in  $\text{ft/s}^2$  is given by the function  $a = 2s$ . When  $t = 0$ ,  $v = 1$  ft/s.  
Find the velocity when the object has moved 2 ft from its initial position?

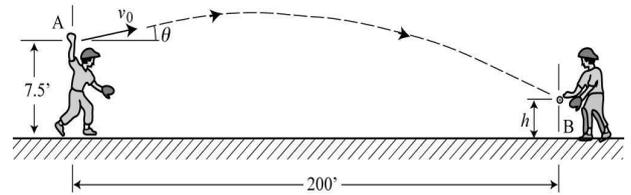
8. ♦♦ During takeoff an airplane starts from rest and accelerates according to  $a = a_0 - kv^2$ , where  $a_0$  is the constant acceleration resulting from the engine thrust and  $-kv^2$  is the acceleration due to aerodynamic drag. If  $a_0 = 2 \text{ m/s}^2$ ,  $k = 0.00004 \text{ m}^{-1}$ , and  $v$  is in meters per second, determine the length of runway required to reach the takeoff speed of 250 km/h if the drag term is (a) excluded and (b) included.

*Ans.* (a)  $s = 1206$  m, (b)  $s = 1269$  m  
Repeat the problem when  $k$  is twice as large.



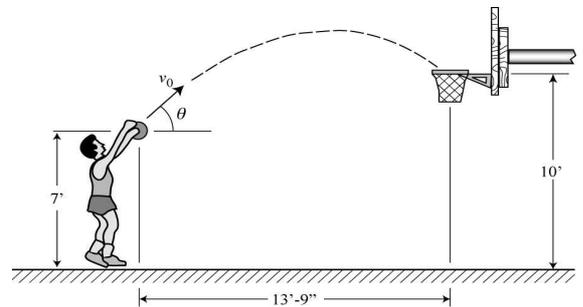
## Curvilinear Motion

9. ♦ An outfielder experiments with two different trajectories for throwing to home plate from the position shown: (a)  $v_0 = 140$  ft/sec with  $\theta = 8^\circ$  and (b)  $v_0 = 120$  ft/sec with  $\theta = 12^\circ$ . For each set of initial conditions, determine the time  $t$  required for the baseball to reach home plate and the elevation  $h$  as the ball crosses the plate.

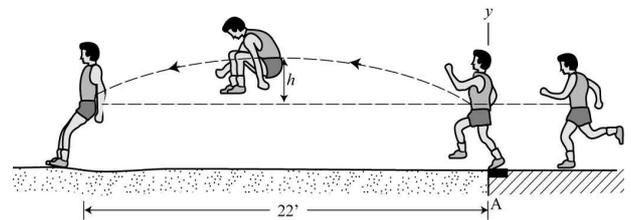


*Ans.* (a)  $t = 1.443$  sec,  $h = 2.10$  ft  
 (b)  $t = 1.704$  sec,  $h = 3.27$  ft

10. ♦ A 7 ft tall basketball player likes to release his shots at an angle  $\theta = 60^\circ$  to the horizontal. The basket is 10' high and he is positioned 13ft and 9 inches from the basket. What initial speed  $v_0$  will cause the ball to pass through the center of the rim?



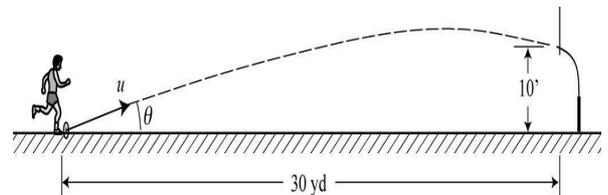
11. ♦ A jumper approaches his takeoff board A with a horizontal velocity of 30 ft/sec. Determine the vertical component  $v_y$  of the velocity of his center of gravity at takeoff for him to make the jump shown. What is the maximum vertical rise  $h$  of his center of gravity?



*Ans.*  $v_y = 11.81$  ft/sec,  $h = 2.16$  ft

What would be the jumping length if the vertical velocity increases by 10% and the horizontal velocity decreases by 10%.

12. ♦♦ A football player attempts a 30-yd field goal. If he is able to impart a velocity  $u$  of 100 ft/sec to the ball, compute the minimum angle  $\theta$  for which the ball will clear the crossbar. (Hint: Let  $m = \tan \theta$ .)



*Ans.*  $\theta = 14.91^\circ$

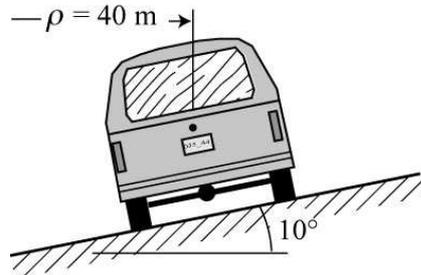
Can you also define the maximum angle?

## Curvilinear Coordinates

13. A minivan accelerates from rest on the road whose constant radius of curvature is 40 m and whose bank angle is  $10^\circ$ . The motion occurs in a horizontal plane. If the constant forward acceleration of the minivan is  $1.8 \text{ m/s}^2$ , determine the magnitude  $a$  of its total acceleration 5 s after starting.

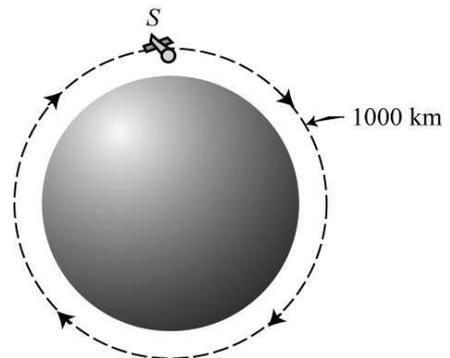
*Ans.*  $a = 2.71 \text{ m/s}^2$

Is the acceleration twice as large at a time  $t = 10\text{s}$ ?



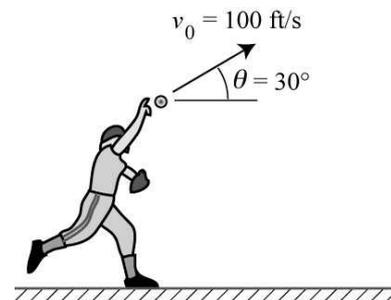
14. ♦ A spacecraft  $S$  is orbiting Jupiter in a circular path 1000 km above the surface with a constant speed. Using Newton's gravitation law,  $F = GMm/R^2$ , calculate the magnitude  $v$  of its orbital velocity with respect to Jupiter. The diameter of Jupiter is 142,984 km and its surface-level gravitational acceleration is  $24.85 \text{ m/s}^2$ .

*Ans.*  $v = 41,900 \text{ m/s}$



15. ♦♦ For the baseball problem sketched below determine the radius of curvature  $\rho$  of the path and the time rate of change  $\dot{v}$  of the speed at times  $t = 1 \text{ sec}$  and  $t = 2.5 \text{ sec}$ , where  $t = 0$  is the time of release from the player's hand.

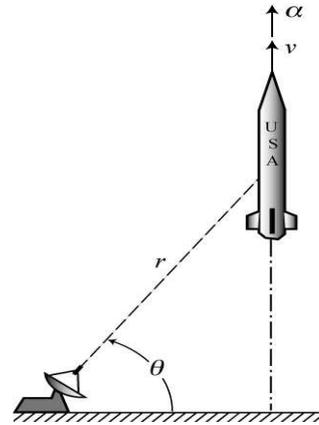
*Ans.*  $\rho = 248 \text{ ft}$ ,  $\dot{v} = -6.48 \text{ ft/sec}^2$   
 $\rho = 278 \text{ ft}$ ,  $\dot{v} = 10.70 \text{ ft/sec}^2$



## Polar Coordinates

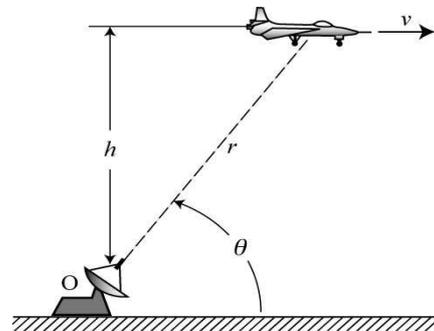
16. A rocket is fired vertically and tracked by the radar shown. When  $\theta$  reaches  $60^\circ$ , other corresponding measurements give the values  $r = 30,000$  ft,  $\dot{r} = 70$  ft/sec<sup>2</sup>, and  $\dot{\theta} = 0.02$  rad/sec. Calculate the magnitudes of the velocity and acceleration of the rocket at this position.

*Ans.*  $v = 1200$  ft/sec,  $a = 67.0$  ft/sec<sup>2</sup>  
Repeat this problem at  $\theta = 70^\circ$ .



17. ♦ A jet flying at a constant speed  $v$  at an altitude  $h = 10$  km is being tracked by radar located at  $O$  directly below the line of flight. If the angle  $\theta$  is decreasing at the rate of  $0.020$  rad/s when  $\theta = 60^\circ$ , determine the value of  $\dot{r}$  at this instant and the magnitude of the velocity  $v$  of the plane.

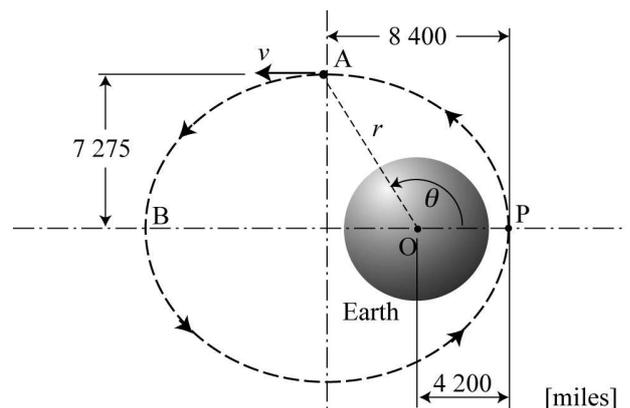
*Ans.*  $\dot{r} = 4.62$  m/s<sup>2</sup>,  $v = 960$  km/h



18. ♦♦ An earth satellite traveling in the elliptical orbit shown has a velocity  $v = 12,149$  mi/hr as it passes the end of the semi minor axis at  $A$ . The acceleration of the satellite at  $A$  is due to gravitational attraction and is  $32.23[3959/8400]^2 = 7.159$  ft/sec<sup>2</sup> directed from  $A$  to  $O$ . For position  $A$  calculate the values of  $\dot{r}$ ,  $\ddot{r}$ ,  $\dot{\theta}$ , and  $\ddot{\theta}$ .

*Ans.*  $\dot{r} = 8910$  ft/sec  
 $\ddot{r} = -1.790$  ft/sec<sup>2</sup>  
 $\dot{\theta} = 3.48(10^{-4})$  rad/sec  
 $\ddot{\theta} = -1.398(10^{-7})$  rad/sec<sup>2</sup>

Determine the gravitational acceleration and repeat the calculations at point  $B$ .

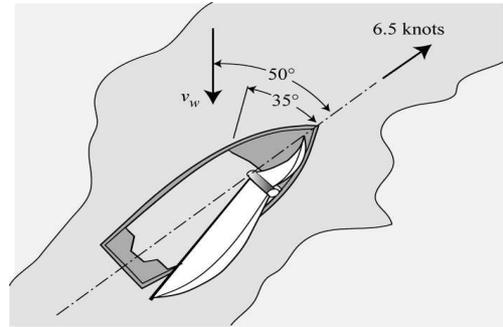


## Relative Motion

19. ♦ A sailboat is moving in the N-E direction against a north wind. The log registers a hull speed of 6.5 knots. A “telltale” (light string tied to the rigging) indicates that the direction of the apparent wind is  $35^\circ$  from the centerline of the boat. What is the true wind velocity  $v_w$ ?

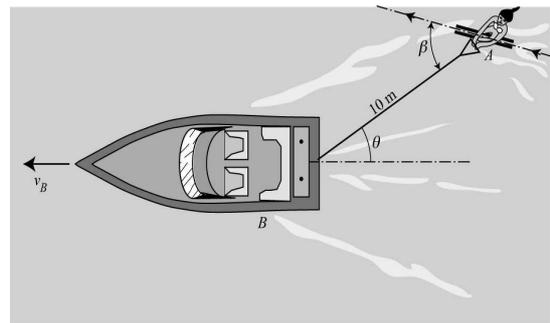
*Ans.*  $v_w = 14.40$  knots

How long does it take the boat to travel a distance of 10 km?



20. ♦♦ To increase his speed, the water skier A cuts across the wake of the tow boat B, which has a velocity of 60 km/h. At the instant when  $\theta = 30^\circ$ , the actual path of the skier makes an angle  $\beta = 50^\circ$  with the tow rope. For this position determine the velocity  $v_A$  of the skier and the value of  $\dot{\theta}$ .

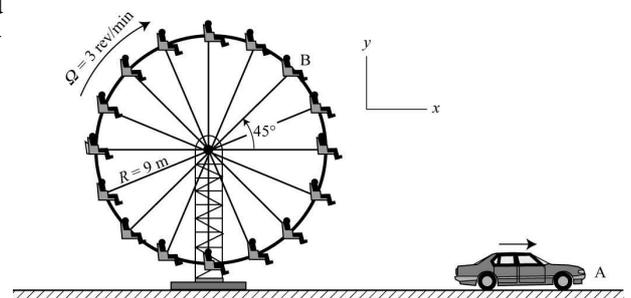
*Ans.*  $v_A = 80.8$  km/h,  $\dot{\theta} = 0.887$  rad/s



21. ♦ The car A has a forward speed of 18 km/h and is accelerating a  $3 \text{ m/s}^2$ . Determine the velocity and acceleration of the car relative to observer B, who rides in a nonrotating chair on the ferris wheel. The angular velocity of the ferris wheel is constant at  $\Omega = 3 \text{ rev/min}$ .

*Ans.*  $\mathbf{v}_{A/B} = 3.00\mathbf{i} + 2.00\mathbf{j} \text{ m/s}$

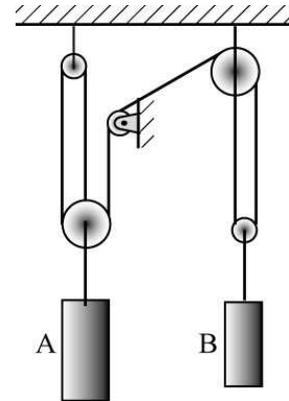
$\mathbf{a}_{A/B} = 3.63\mathbf{i} + 0.628\mathbf{j} \text{ m/s}^2$



## Constrained Motion

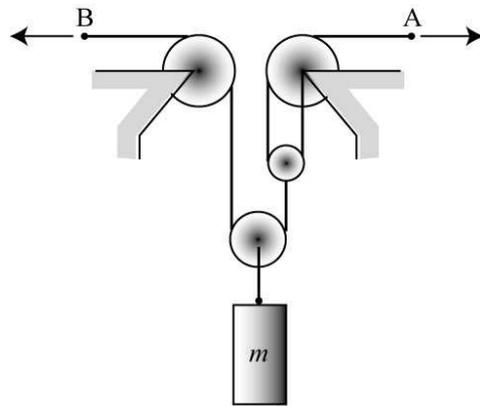
22. At a certain instant, cylinder  $A$  has a downward velocity of  $0.8 \text{ m/s}$  and an upward acceleration of  $2 \text{ m/s}^2$ . Determine the corresponding velocity and acceleration of cylinder  $B$ .

*Ans.*  $v_B = 1.2 \text{ m/s}$  up  
 $a_B = 3 \text{ m/s}^2$  down



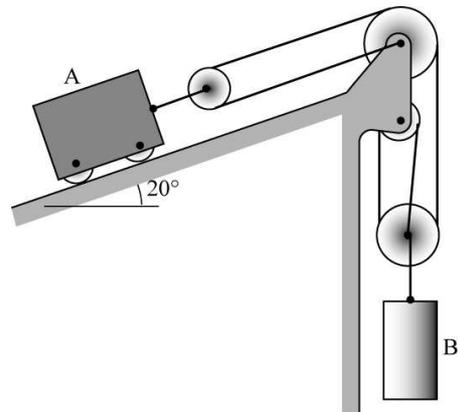
23. For the pulley system shown, each of the cables at  $A$  and  $B$  is given a velocity of  $2 \text{ m/s}$  in the direction of the arrow. Determine the upward velocity  $v$  of the load  $m$ .

*Ans.*  $v = 1.5 \text{ m/s}$   
 If  $v_B = 2 \text{ m/s}$ , determine  $v_A$  such that the mass  $m$  does not move.



24. ♦ Cylinder  $B$  has a downward velocity of  $2 \text{ ft/sec}$  and an upward acceleration of  $0.5 \text{ ft/sec}^2$ . Calculate the velocity and acceleration of block  $A$ .

*Ans.*  $v_A = 3 \text{ ft/sec}$  up,  $a_A = 0.75 \text{ ft/sec}^2$  down



# DYNAMICS

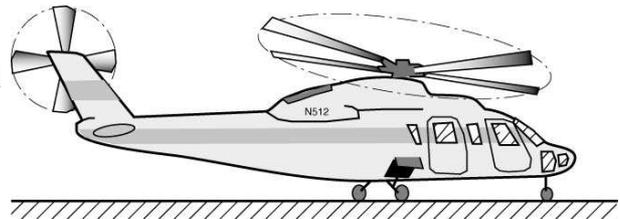
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## PROBLEMS B

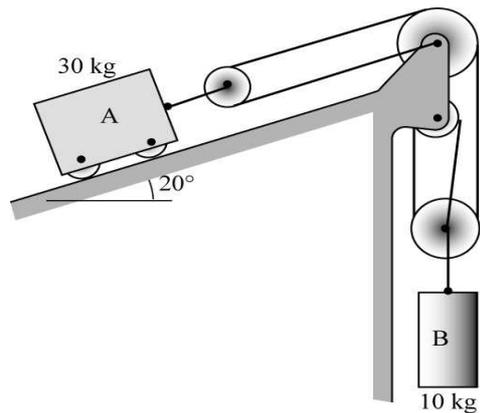
### KINETICS OF PARTICLES

#### Rectilinear Motion

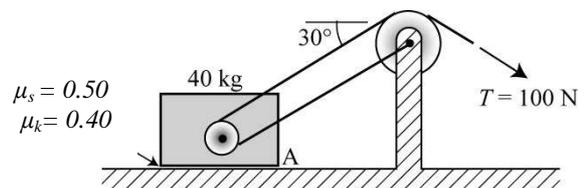
1. If the 15,000-lb helicopter starts from rest and its rotor exerts a constant 20,000-lb vertical force, how high does it rise in 5 s?



2. ♦ Neglect all friction and the mass of the pulleys and determine the accelerations of bodies *A* and *B* upon release from rest.  
*Ans.*  $a_A = 1.024 \text{ m/s}^2$  down the incline  
 $a_B = 0.682 \text{ m/s}^2$  up

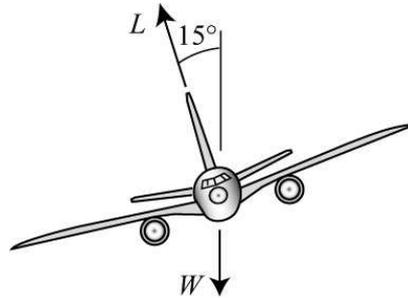


3. ♦♦ Compute the acceleration of block *A* for the instant depicted. The dynamic friction is  $\mu_k = 0.40$ . Neglect the mass of the pulley.  
*Ans.*  $a = 1.406 \text{ m/s}^2$   
 What is the minimal tension in the cable to initiate motion?

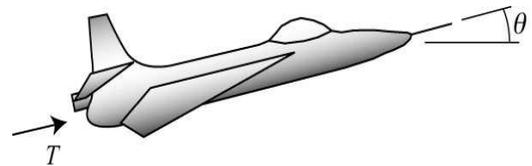


## Curvilinear Motion

4. ♦ An airplane of weight  $W = 200,000$  lb makes a turn at constant altitude and at constant velocity  $v = 630$  ft/s. The bank angle is  $15^\circ$ .
- Determine the lift force  $L$ .
  - What is the radius of curvature of the plane's path?

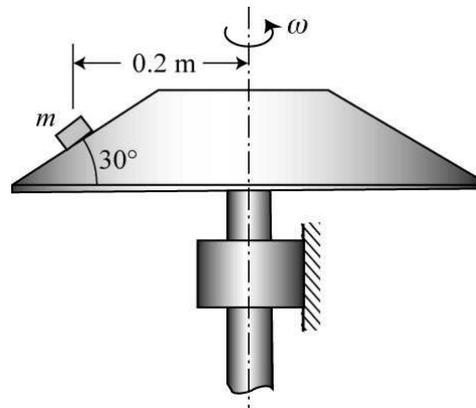


5. ♦ The 30-Mg aircraft is climbing at the angle  $\theta = 15^\circ$  under a jet thrust  $T$  of 180 kN. At the instant shown, its speed is 300 km/h and is increasing at the rate of  $1.96$  m/s<sup>2</sup>. Also  $\theta$  is decreasing as the aircraft begins to level off. If the radius of curvature of the path at this instant is 20 km, compute the lift  $L$  and drag  $D$ . (Lift  $L$  and drag  $D$  are the aerodynamic forces normal to and opposite to the flight direction, respectively.)



*Ans.*  $D = 45.0$  kN,  $L = 274$  kN

6. ♦♦ The small object of mass  $m$  is placed on the rotating conical surface at the radius shown. If the coefficient of static friction between the object and the rotating surface is 0.8, calculate the maximum angular velocity  $\omega$  of the cone about the vertical axis for which the object will not slip. Assume very gradual changes in angular velocity.

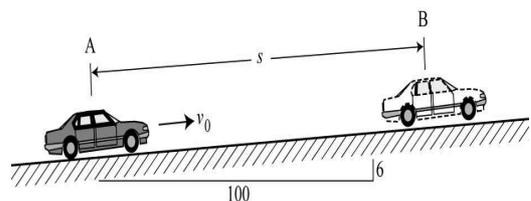


*Ans.*  $\omega = 2.73$  rad/s

Does the critical velocity increase linearly with the friction coefficient?

## Work-Energy

7. ♦ The car is moving with a speed  $v_o = 65$  mi/hr up the 6-percent grade and the driver applies the brakes at point A, causing all wheels to skid. The coefficient of kinetic friction on the wet pavement  $\mu_k = 0.6$ . Determine the stopping

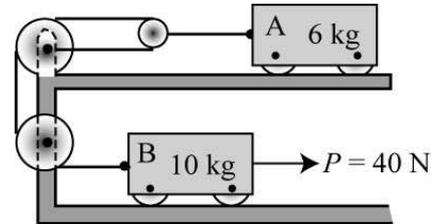


distance  $s_{AB}$ . Repeat your calculations for the case when the car is moving downhill from  $B$  to  $A$ .

*Ans.*  $s_{AB} = 214 \text{ ft}$ ,  $s_{BA} = 262 \text{ ft}$

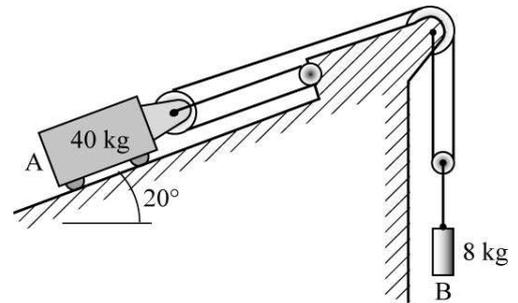
8. The force  $P = 40 \text{ N}$  is applied to the system, which is initially at rest. Determine the speeds of  $A$  and  $B$  after  $A$  has moved  $0.4 \text{ m}$ .

*Ans.*  $v_A = 1.180 \text{ m/s}$ ,  $v_B = 2.36 \text{ m/s}$



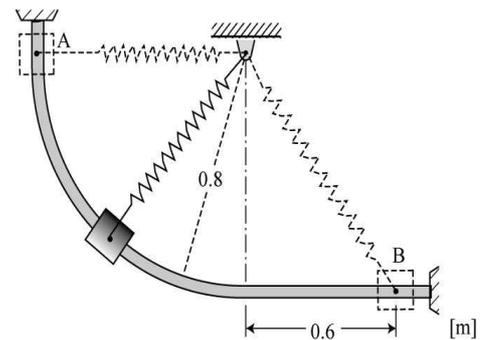
9. ♦ If the system is released from rest, determine the speed of both masses after  $B$  moved  $1 \text{ m}$ . Neglect friction and the masses of the pulleys.

*Ans.*  $v_A = 0.616 \text{ m/s}$ ,  $v_B = 0.924 \text{ m/s}$



10. ♦ The spring has an unstretched length of  $0.4 \text{ m}$  and a stiffness of  $200 \text{ N/m}$ . The  $3\text{-kg}$  slider and attached spring are released from rest at  $A$  and move in the vertical plane. Calculate the velocity  $v$  of the slider as it reaches  $B$  in the absence of friction.

*Ans.*  $v = 1.537 \text{ m/s}$

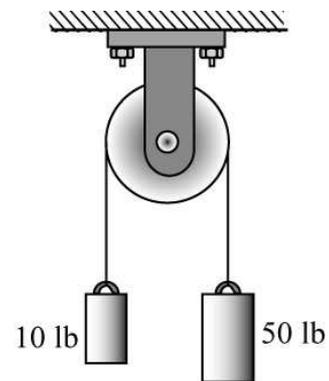


## Linear Momentum

11. A boat is going 50 mi/hr when its motor is suddenly turned off. Its velocity decreases to 20 mi/hr within 5 s. The boat and its passengers weigh 1800 lb. Determine the magnitude of the average force exerted on the boat by hydrodynamic and aerodynamic drag during the 5 s.

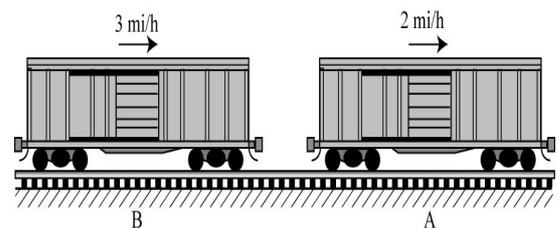


12. ♦ The two weights are released from rest. Apply the principle of impulse and momentum to each weight individually. What is the magnitude of their velocity after one second?



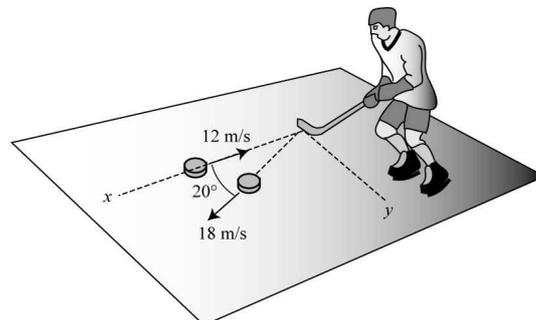
## Impact

13. ♦ Freight car A with a gross weight of 150,000 lb is moving along the horizontal track in a switching yard at 2 mi/hr. Freight car B with a gross weight of 120,000 lb and moving a 3 mi/hr overtakes car A and is coupled to it. Determine (a) the common velocity  $v$  of the two cars as they move together after being coupled and (b) the loss of energy  $|\Delta E|$  due to the impact.



Ans. (a)  $v = 2.44$  mi/hr, (b)  $|\Delta E| = 2230$  ft-lb

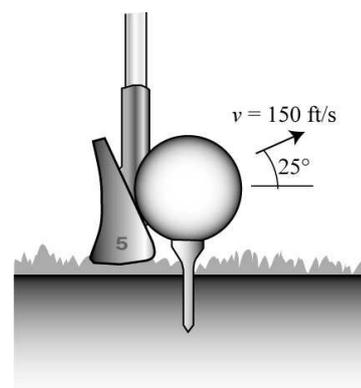
14. ♦ The hockey puck with a mass of 0.20 kg has a velocity of 12 m/s before being struck by the hockey stick. After the impact the puck moves in the new direction shown with a velocity of 18 m/s. If the stick is in contact with the puck for 0.04 s, compute the magnitude of the average force  $\mathbf{F}$  exerted by the stick on the puck during contact, and find the angle  $\beta$  made by  $\mathbf{F}$  with the  $x$ -direction.



*Ans.*  $\mathbf{F} = 147.8 \text{ N}$ ,  $\beta = 12.02^\circ$

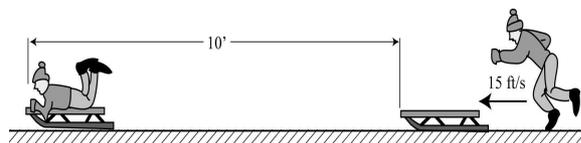
Repeat this problem if the impact duration is 0.1 s.

15. ♦ The 1.62-oz golf ball is struck by the five-iron and acquires the velocity shown in a time period of 0.001 sec. Determine the magnitude  $R$  of the average force exerted by the club on the ball. What acceleration magnitude  $a$  does this force cause and what is the distance  $d$  over which the launch velocity is achieved, assuming constant acceleration?

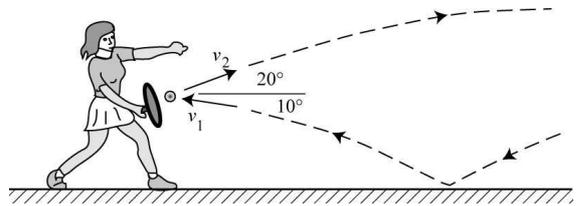


*Ans.*  $R = 472 \text{ lb}$ ,  $a = 150,000 \text{ ft/sec}^2$ ,  $d = 0.9 \text{ in}$

16. ♦♦ A boy weighing 100 lb runs and jumps on his 20-lb sled with a horizontal velocity of 15 ft/sec. If the sled and boy coast 10 ft on the level snow before coming to rest, compute the coefficient of kinetic friction  $\mu_k$  between the snow and the runners of the sled.



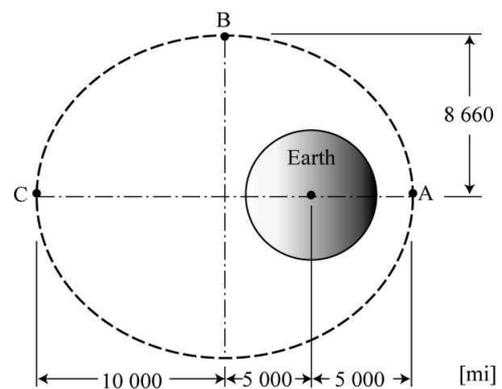
17. ♦♦ A tennis player strikes the tennis ball with her racket while the ball is still rising. The ball speed before impact with the racket is  $v_1 = 15$  m/s and after impact its speed is  $v_2 = 22$  m/s, with directions as shown in the figure. If the 60-g ball is in contact with the racket for 0.05 s, determine the magnitude of the average force  $\mathbf{R}$  exerted by the racket on the ball. Find the angle  $\beta$  made by  $\mathbf{R}$  with the horizontal. Comment on the treatment of the ball weight during impact.



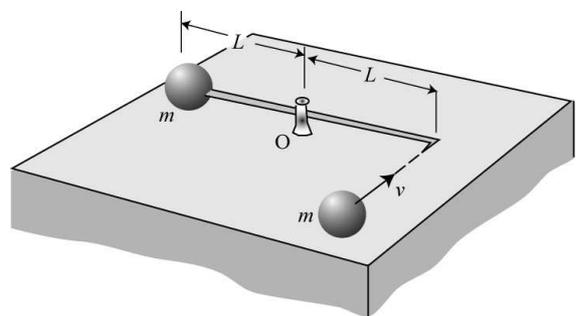
*Ans.*  $\mathbf{R} = 43.0$  N,  $\beta = 8.68^\circ$

## Angular Momentum

18. A satellite is in elliptic orbit around the earth. Its velocity at the perigee A is 28,280 ft/s. What is its velocity at the apogee C.

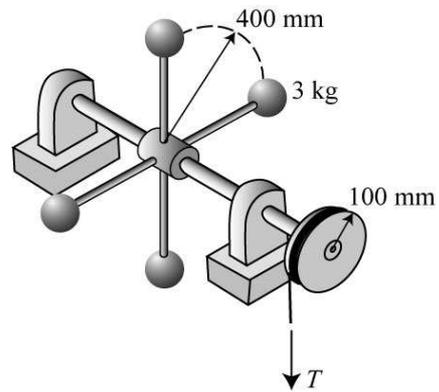


19. ♦ The small sphere of mass  $m$  traveling with speed  $v$  strikes and becomes attached to the end of the stationary assembly that pivots freely about a vertical axis at  $O$ . Determine the angular velocity  $\omega$  of the assembly after impact and calculate the change  $\Delta E$  in the system energy. Neglect the mass of the rod compared with  $m$ . Also neglect any friction.



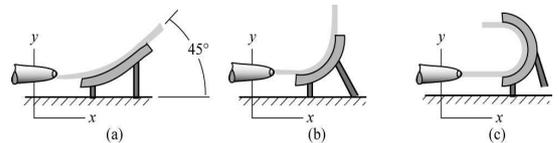
*Ans.*  $\omega = v/2L$ ,  $\Delta E = -1/4 mv^2$

20. ♦ The assembly starts from rest and reaches an angular speed of 150 rev/min under the action of a 20-N force  $T$  applied to the string for  $t$  seconds. Determine  $t$ . Neglect friction and all masses except those of the four 3-kg spheres, which may be treated as particles.  
*Ans.*  $t = 15.08$  s



### Mass Flow

21. ♦ A nozzle ejects a stream of water horizontally at 40 mi/hr with a volumetric flow rate of  $30 \text{ m}^3/\text{s}$ , and the stream is deflected in the horizontal plane by a plate. Determine the force exerted on the plate by the stream in cases (a), (b) and (c).



22. ♦ The jet ski has reached its maximum velocity of 70 km/h when operating in salt water at a mass density  $\rho = 1030 \text{ kg/m}^3$ . The water intake is in the horizontal tunnel in the bottom of the hull, so the water enters the intake at the velocity of 70 km/h relative to the ski. The motorized pump discharges water from the horizontal exhaust nozzle of 50-mm diameter at the rate of  $0.082 \text{ m}^3/\text{s}$ . Calculate the resistance  $R$  of the water to the hull at the operating speed.  
*Ans.*  $R = 1,885$  N



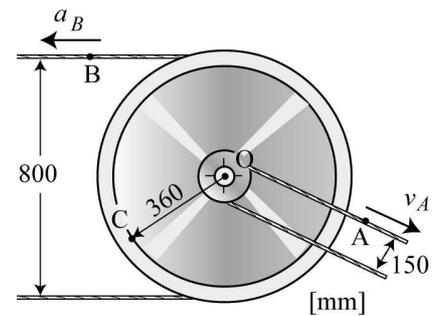
# DYNAMICS

Pierre Julien

## PROBLEMS C

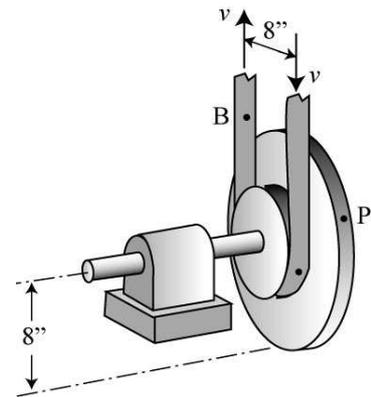
### RIGID BODY KINEMATICS

- ◆ The two V-belt pulleys form an integral unit and rotate about the fixed axis at  $O$ . At a certain instant, point  $A$  on the belt of the smaller pulley has a velocity  $v_A = 1.5 \text{ m/s}$ , and point  $B$  on the belt of the larger pulley has an acceleration  $a_B = 45 \text{ m/s}^2$  as shown. For this instant determine the magnitude of the acceleration  $a_C$  of point  $C$  and sketch the vector in your solution. Also calculate the acceleration of  $A$ .



- ◆ The two attached pulleys are driven by the belt with increasing speed. When the belt reaches a speed  $v = 2 \text{ ft/sec}$ , the total acceleration of point  $P$  is  $26 \text{ ft/sec}^2$ . For this instant determine the angular acceleration  $\alpha$  of the pulleys and the acceleration of point  $B$  on the belt.

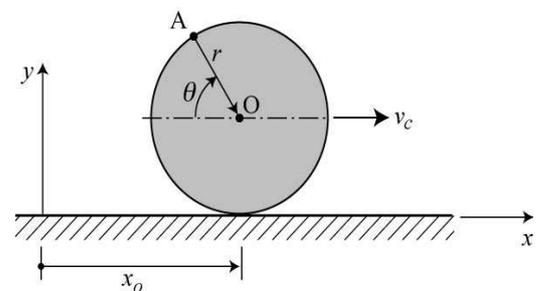
Ans.  $\alpha = 15 \text{ rad/sec}^2, a_B = 5 \text{ ft/sec}^2$



### Absolute Motion

- ◆◆◆ The wheel of radius  $r$  rolls without slipping and its center  $O$  has a constant velocity  $v_O$  to the right. Determine expressions for the velocity  $v$  and acceleration  $a$  of point  $A$  on the rim by differentiating its  $x$ - and  $y$ -coordinates. Represent your result graphically as vectors on your sketch and show  $v$  is the vector sum of two  $v_O$  vectors.

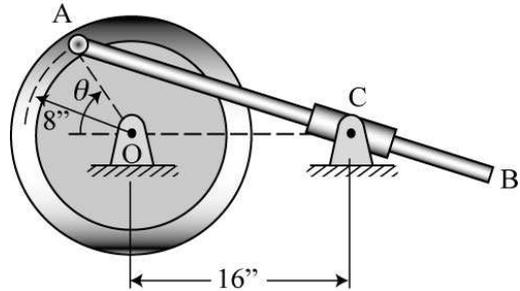
Ans.  $v = v_O \sqrt{2(1 + \sin \theta)}, a = \frac{v_O^2}{r}$  toward  $O$



## Relative Velocity

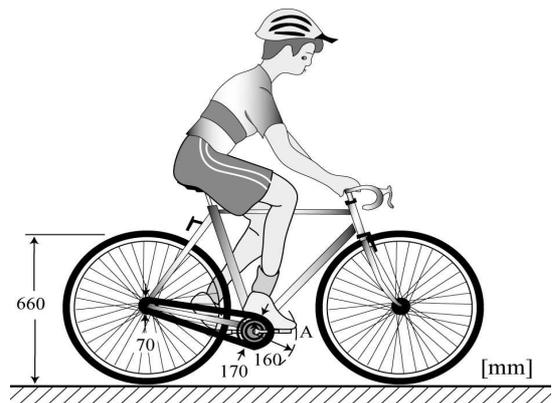
4. ♦ The flywheel turns clockwise with a constant speed of 600 rev/min. the connecting link  $AB$  slides through the pivoted collar at  $C$ . Calculate the angular velocity  $\omega$  of  $AB$  for the instant when  $\theta = 60^\circ$ .

*Ans.*  $\omega = 17.95$  rad/sec CW

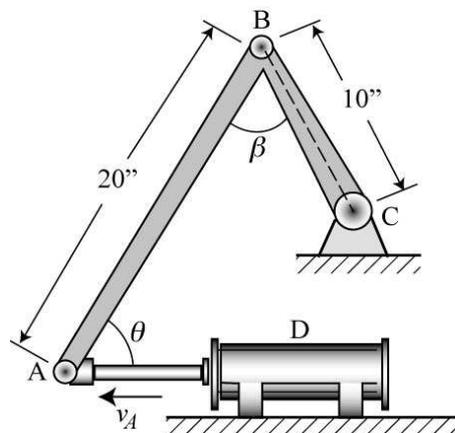


5. ♦♦ The rider of the bicycle shown pumps steadily to maintain a constant speed of 16 km/h against a slight head wind. Calculate the maximum and minimum magnitudes of the absolute velocity of the pedal at a radius 160 mm from the pedal axle. The diameter of the wheel is 660 mm, the rear sprocket wheel 70 mm and the pedal sprocket wheel 170 mm. The radius of the pedal is 160 mm.

*Ans.*  $(v_A)_{max} = 5.33$  m/s  
 $(v_A)_{min} = 3.56$  m/s



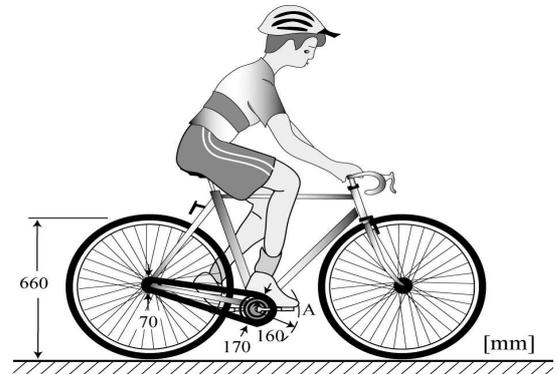
6. ♦ For an interval of its motion the piston rod of the hydraulic cylinder has a velocity  $v_A = 4$  ft/sec as shown. At a certain instant  $\theta = \beta = 60^\circ$ , determine the angular velocity  $\omega_{BC}$  of link  $BC$ .



## Relative Acceleration

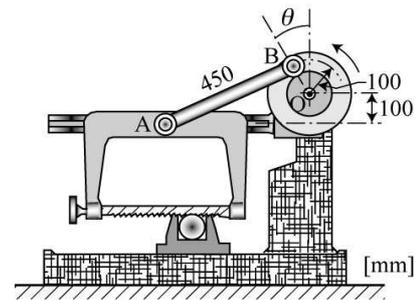
7. ♦♦ If the cyclist starts from rest and acquires a velocity of 2 m/s in a distance of 3 m with constant acceleration, compute the magnitude of the acceleration of pedal for this condition if it is in the top position.

Ans.  $|\vec{a}_p| = 1.28 \text{ m/s}^2$



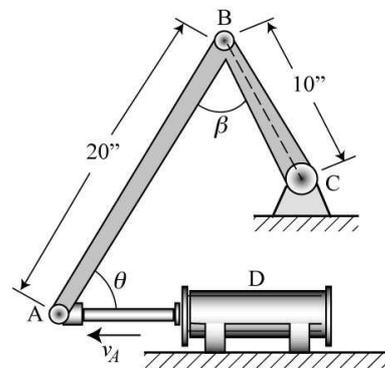
8. ♦♦ The elements of a power hacksaw are shown in the figure. The saw blade is mounted in a frame which slides along the horizontal guide. If the motor turns the flywheel at a constant counterclockwise speed of 60 rev/min, determine the acceleration of the blade for the position where  $\theta = 90^\circ$ , and find the corresponding angular acceleration of the link AB.

Ans.  $a_A = 4.89 \text{ m/s}^2$ ,  $a_{AB} = 0.467 \text{ rad/s}^2$  CCW  
As an exam question, could you solve this for  $\theta = 0$ ?



9. ♦♦♦ For the instant when  $\theta = \beta = 60^\circ$ , the hydraulic cylinder gives A a velocity  $v_A = 4$  ft/sec which is increasing by 3 ft/sec each second. For this instant determine the angular acceleration of link BC.

Ans.  $\alpha_{BC} = 2.08 \text{ rad/sec}^2$  CCW  
Can you also find  $\omega_{BC}$ ?

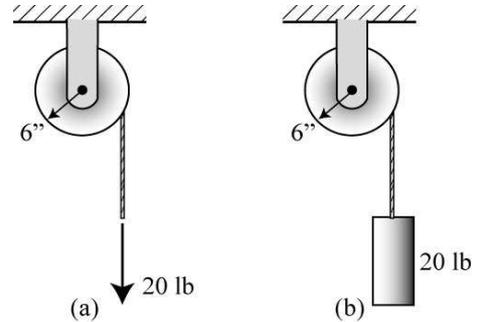


## Rigid Body Kinetics

10. The moment of inertia of the pulley is  $0.4 \text{ slug-ft}^2$ . Determine the pulley's angular acceleration and the tension in the cable of the two cases.

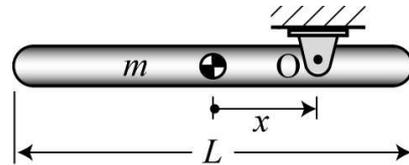
Ans.  $\alpha_A = 25 \text{ rad/s}^2$   
 $\alpha_B = 18 \text{ rad/s}^2$

Can you explain why the answers are different?



11. ♦♦♦ Here is a drumstick problem. For what value of  $x$  is the horizontal bar's angular acceleration a maximum, and what is the maximum angular acceleration?

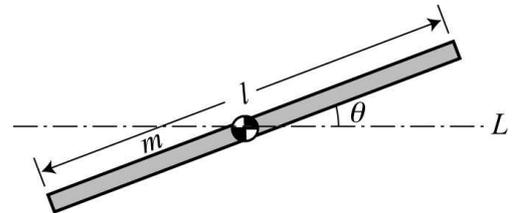
Ans.  $x = L/\sqrt{12}$   
 $\alpha = \frac{12gx}{L^2 + 12x^2}$



## Moment of Inertia

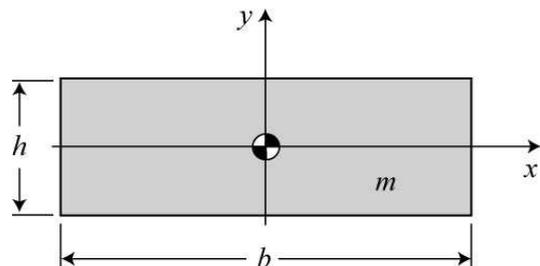
12. ♦ The homogeneous, slender bar has mass  $m$  and length  $l$ . Use integration to determine the moment of inertia of the bar about the axis  $L$ .

Ans.  $I_L = \frac{ml^2}{12} \sin^2 \theta$



13. ♦ The homogeneous, thin plate is of uniform thickness and mass  $m$ . Determine its moments of inertia about the  $x$ ,  $y$  and  $z$  axes.

Ans.  $I_y = \frac{mb^2}{12}$   
 $I_z = \frac{m(b^2 + h^2)}{12}$

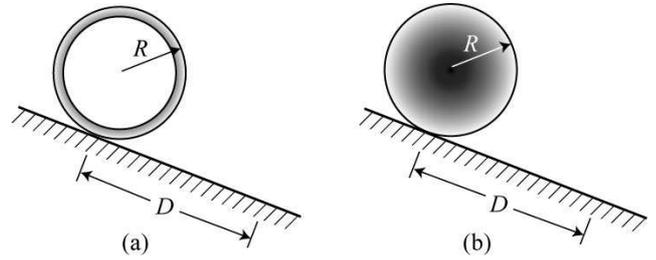


## General Motion

14. ♦ A thin ring and a circular disk, each of mass  $m$  and radius  $R$ , are released from rest on an inclined surface and allowed to roll a distance  $D$ . Determine the angular accelerations for both cases, and the ratio of the times required.

$$\text{Ans. } \alpha_a = \frac{g \sin \theta}{2r}$$

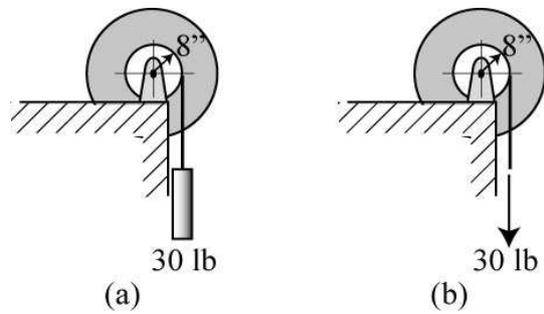
$$t_r = \frac{t_b}{t_a} = \sqrt{\frac{3}{4}}$$



15. Each of the two drums and connected hubs of 8-in. radius weighs 200 lb and has a radius of gyration about its center of 15 in. Calculate the angular acceleration of each drum. Friction in each bearing is negligible.

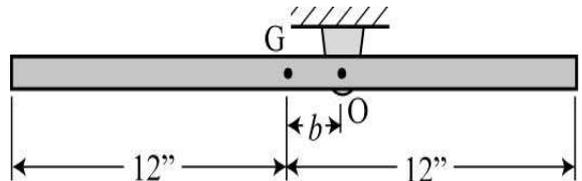
$$\text{Ans. } \alpha_A = 1.976 \text{ rad/sec}^2, \alpha_B = 2.06 \text{ rad/sec}^2$$

What are the mass moments of inertia?



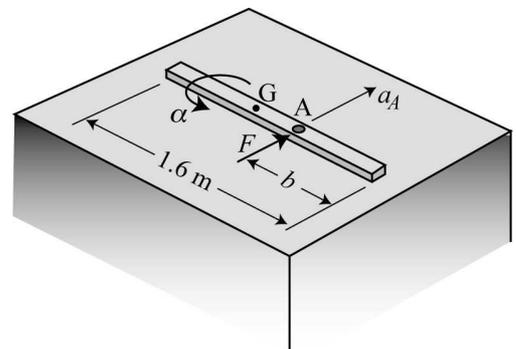
16. ♦♦ The uniform 16.1-lb slender bar is hinged about a horizontal axis through  $O$  and released from rest in the horizontal position. Determine the distance  $b$  from the mass center to  $O$  which will result in an initial angular acceleration of  $16.1 \text{ rad/sec}^2$ , and find the force  $R$  on the bar at  $O$  just after release.

$$\text{Ans. } b = 2.20 \text{ in.}, R = 14.62 \text{ lb}$$



17. ♦♦ The uniform slender bar rests on a smooth horizontal surface when a force  $F$  is applied normal to the bar at point  $A$ . Point  $A$  is observed to have an initial acceleration  $a_A$  of  $20 \text{ m/s}^2$ , and the bar has a corresponding angular acceleration  $\alpha$  of  $18 \text{ rad/s}^2$ . Determine the distance  $b$ .

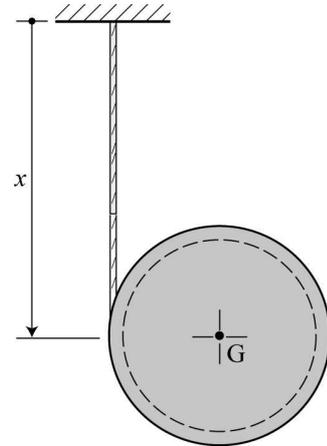
$$\text{Ans. } b = 0.553 \text{ m}$$



18. ♦ A long cable length  $L$  and mass  $\rho$  per unit length is wrapped around the periphery of a spool of negligible mass. One end of the cable is fixed and the spool is released from rest in the position shown. Find the initial acceleration  $a$  of the center of the spool.

*Ans.*  $a = g/2$

Can you explain why the acceleration is less than  $g$ ?

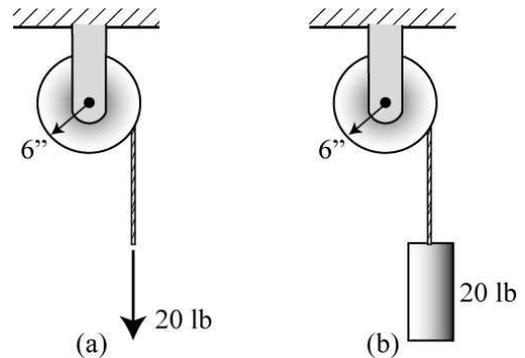


## Work Energy

19. ♦ The moment of inertia of the pulley is  $0.4 \text{ slug-ft}^2$ . The pulley starts from rest. For both cases, use the principle of work and energy to determine the pulley's angular velocity when it has turned 1 revolution.

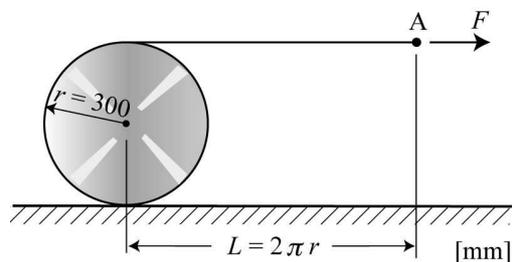
*Ans.*  $\omega_a = 17.7 \text{ rad/s}$

$\omega_b = 15 \text{ rad/s}$



20. ♦ The 100-kg homogeneous cylindrical disk is at rest when the force  $F = 500 \text{ N}$  is applied to a cord wrapped around it, causing the disk to roll. Use the principle of work and energy to determine the disk's angular velocity when it has turned 1 revolution. Notice here that when the drum completes one revolution, point A will move  $4\pi r$  to the right!

*Ans.*  $\omega = 16.7 \text{ rad/s}$

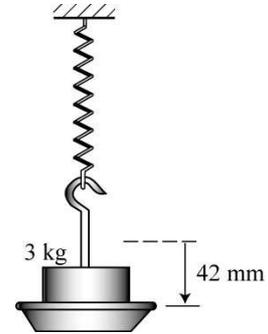


## VIBRATIONS

### Free Vibrations

21. When a 3-kg collar is placed upon the pan which is attached to the spring of unknown constant, the additional static deflection of the pan is observed to be 42 mm. Determine the spring constant  $k$  in N/m, lb/in., and lb/ft.

*Ans.*  $k = 701$  N/m,  $k = 4.00$  lb/in.  
and  $k = 48.0$  lb/ft

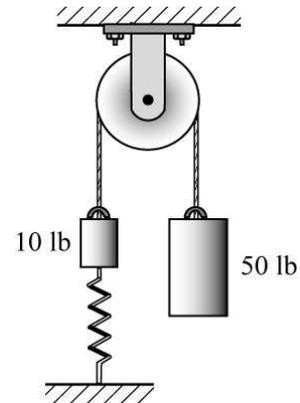


22. ♦ If the spring constant is  $k = 30$  lb/ft and the moment of inertia of the pulley is negligible, what is the frequency of vertical vibrations of the weights relative to their equilibrium positions?

*Ans.*  $\omega_n = 4$  rad/s

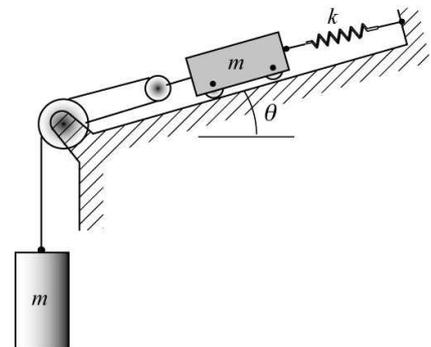
$$f = \frac{2}{\pi s}$$

As a review problem, can you also calculate the maximum vertical displacement, the maximum velocity and the maximum tension in the spring?



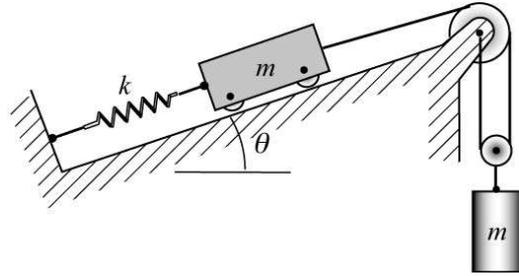
23. ♦ Calculate the natural circular frequency  $\omega_n$  of the system shown in the figure. The mass and friction of the pulleys are negligible.

*Ans.*  $\omega_n = \sqrt{k/5m}$



24. Calculate the natural frequency  $\omega_n$  of the system shown in the figure. The mass and friction of the pulleys are negligible.

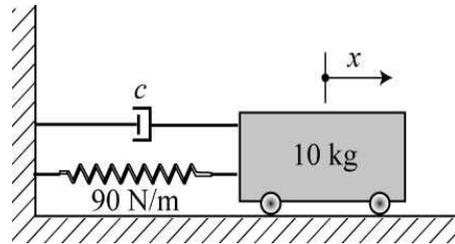
*Ans.*  $\omega_n = \sqrt{4k / 5m}$



### Damped Vibrations

25. ♦ The damping constant of the damped spring-mass oscillator is  $c = 20 \text{ N-s/m}$ . What are the period and natural frequency of the system? Compare them to the period and natural frequency when the system is undamped.

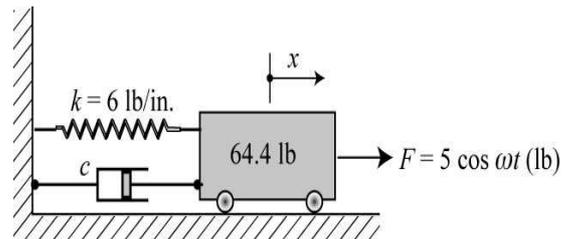
*Ans.*  $f_d = 0.45/\text{s}$   
 $\tau_d = 2.22 \text{ s}$



### Forced Vibrations

26. ♦♦ The 64.4-lb cart is acted upon by the harmonic force shown in the figure. If  $c = 0$ , determine the range of the driving frequency  $\omega$  for which the amplitude of the steady-state response is less than 3.5 in.

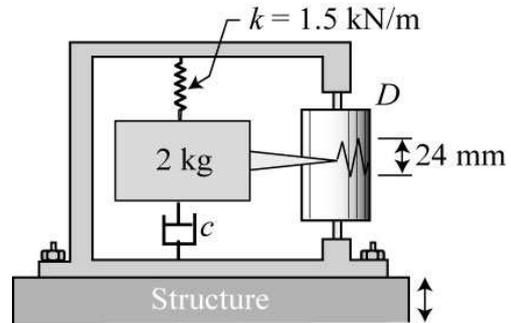
*Ans.*  $\omega < 5.24 \text{ rad/sec}$  or  $\omega > 6.68 \text{ rad/sec}$



*These last two problems are most interesting but optional*

27. ♦♦ The seismic instrument is mounted on a structure which has a vertical vibration with a frequency of 5 Hz and a double amplitude of 18 mm. The sensing element has a mass  $m = 2$  kg, and the spring stiffness is  $k = 1.5$  kN/m. The motion of the mass relative to the instrument base is recorded on a revolving drum and shows a double amplitude of 24 mm during the steady-state condition. Calculate the viscous damping constant  $c$ .

*Ans.*  $c = 44.6$  N·s/m



28. ♦♦ Determine the amplitude of vertical vibration of the spring-mounted trailer as it travels at a velocity of 25 km/h over a bumpy road whose surface may be expressed by a sine or cosine term. The mass of the trailer is 500 kg and that of the wheels alone may be neglected. During the loading each 75 kg added to the load caused the trailer to sag 3 mm on its springs. Assume that the wheels are in contact with the road at all times and neglect damping. At what critical speed  $v_c$  is the vibration of the trailer greatest?

*Ans.*  $X = 14.75$  mm,  $v_c = 15.23$  km/h

