

## Consolidate and Debrief

### Key Concepts

- A combination is a set of items taken from another set in which order does not matter. In a permutation, the order of the items matters.
- The number of combinations of  $r$  items taken from a set of  $n$  items is

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

### Reflect

- R1.** Five people are chosen from a group of eight people. Describe a situation for this set that involves
- permutations
  - combinations
- R2.** Describe two everyday situations in which items are chosen and the order of the selections does not matter.
- R3.** Which situation has a greater number of possibilities, one in which order matters or one in which order does not matter? Explain why.

### Practise

Choose the best answer for #2 and #3.

1. Convert to factorial form, then evaluate.

- |                               |  |
|-------------------------------|--|
| a) ${}_9 C_5$                 | b) ${}_8 C_4$                              |
| c) $C(12,3)$                  | d) $\binom{11}{5}$                         |
| e) ${}_7 C_2 \times {}_6 C_3$ | f) $\binom{101}{98} \times \binom{101}{3}$ |

2. Which is an incorrect way of writing  ${}_{10} C_3$ ?

- |                            |                            |
|----------------------------|----------------------------|
| A $\frac{{}_{10} P_3}{3!}$ | B $\frac{{}_{10} P_3}{7!}$ |
| C $\frac{10!}{7!3!}$       | D $\frac{10!}{3!7!}$       |

3. How many three-member committees can be formed from a group of nine people?

- |       |       |
|-------|-------|
| A 27  | B 84  |
| C 504 | D 729 |

4. In how many ways could 6 online magazine subscriptions be chosen from a set of 10 magazines?

5. In how many ways could you choose 4 packages of pasta from a bin containing 11 different packages of pasta?
6. In how many ways could you reach into a bag containing 10 marbles and pull out none of them?

7. Refer to #6. Evaluate each combination by first writing in factorial form. Remember,  $0! = 1$ .

- |               |                  |
|---------------|------------------|
| a) ${}_1 C_0$ | b) ${}_2 C_0$    |
| c) ${}_3 C_0$ | d) ${}_{15} C_0$ |
| e) ${}_n C_0$ |                  |

### Apply

8. **Application** On an English exam, students need to answer six out of eight questions in Part A and two out of four questions in Part B. The order in which they answer the questions does not matter. In how many ways could a student answer the questions on this exam?

9. From a standard deck, how many five-card hands contain the following?
- only black cards
  - all face cards
  - no hearts
  - two red and three black cards
  - one face card

10. **Communication** Juries are chosen from large pools of people selected at random from the local population. A jury pool has 40 people.



- How many ways are there to form a 12-person jury in a criminal case?
  - How many ways are there to form a 6-person jury in a civil case?
  - Which situation gives a larger number of ways? Explain why this is to be expected.
11. A dealership has six models of trucks and five models of cars for sale. Wayne sells four vehicles this week. How many of the following combinations of four can be formed?
- no restrictions
  - two trucks and two cars
  - three cars and one truck
  - only cars
  - only trucks
  - How are the answers to parts b) to e) related to part a)? Explain why this is true.

12. **Communication** Fourteen family members have a one-on-one video chat with each other to wish each person happy new year.

- Is this situation a permutation or a combination? Explain.
- How many video chats will occur? Express your answer using standard combinatorial notation and then evaluate.

13. Ten points are drawn on the circumference of a circle. Using these points as vertices,

- how many quadrilaterals can be drawn?
- how many pentagons can be drawn?
- how many polygons of  $n$  sides can be drawn?

14. **Thinking**

- Compare each pair of values.
  - ${}_7C_2$  and  ${}_7C_5$
  - ${}_4C_3$  and  ${}_4C_1$
  - ${}_{12}C_4$  and  ${}_{12}C_8$
- State a hypothesis from your observations in part a). Explain why this makes sense when you are thinking about groups.
- Prove your hypothesis algebraically, using  ${}_nC_r$  in your proof.

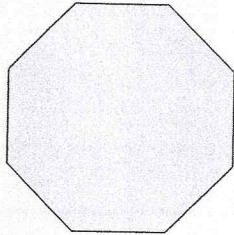
15. **Communication** In a drama class of 18 students, nine are selected to be actors in a play, five will build sets, and four will be stage hands. In how many ways could the class be divided up?

- Make your calculations by selecting the actors first.
- Make your calculations by selecting the set builders first.
- Compare your answers. Explain the results.

16. a) How many diagonals are there in any convex octagon?

**Literacy Link**

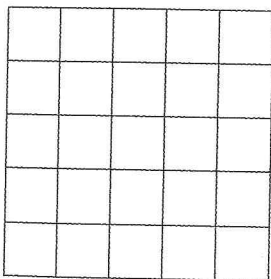
Convex means curved out. Concave means curved in. A convex octagon has all vertices pointing outward.



- b) How many diagonals are there in a convex polygon with  $n$  sides? Explain your reasoning.

✓ **Achievement Check**

17. Ten identical playing pieces are placed on a 5 by 5 game board.



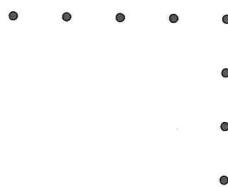
- a) In how many ways could 10 playing pieces be placed on the board if there are no restrictions?
- b) In how many ways could 10 playing pieces be placed on the board if there must be two pieces in each row?
- c) Describe how the results would change if the playing pieces were all different.
18. In how many ways can 15 people be divided into three identical groups of five?
19. In how many ways can a team of 20 hockey players be accommodated in 10 two-person hotel rooms? Assume that the order of assigning the rooms does not matter.

20. Compare your technique in #18 and #19 to the one you used in #12 in section 3.1 on page 109.

21. Show that the number of ways of dividing a class of 30 students into six teams of five members is the same as the number of ways of arranging five red, five green, five purple, five blue, five white, and five black balls.

22. **Communication** For  $r > 0$ , will there always be more  $r$ -permutations of  $n$  items or  $r$ -combinations of  $n$  items? Why?

23. **Thinking** Five points are drawn horizontally, and four points are drawn vertically, with the top one overlapping the point on the right side. How many triangles can be formed using the points as vertices?



**Processes**

**Problem Solving**

How would you solve a simpler version of this problem?

**Extend**

24. a) Show that the product of three consecutive numbers is divisible by 3!.  
b) Show that the product of  $r$  consecutive numbers is divisible by  $r!$ .
25. Solve for  $n$  in  $n! = 12 \times {}_n C_2$ .
26. How many ways are there to choose three numbers from 1 to 20 so that no two are consecutive?