Chapter 2 Review

Learning Goals

Section	After this section, I can
2.1	• make lists, charts, and tree diagrams to organize counting
2.2	• use the fundamental counting principle for counting and to solve problems
2.3	• see how using permutations has advantages over other counting techniques • solve simple problems using techniques for counting permutations • write permutation solutions using proper mathematical notation
2.4	• use the rule of sum to solve counting problems
2.5	• solve probability problems using counting principles for situations with equally likely outcomes

2.1 Organized Counting, pages 64–69

1. Draw a tree diagram showing all the possible outcomes (win, loss, or tie) in three games between two hockey teams. How many possible outcomes are there?

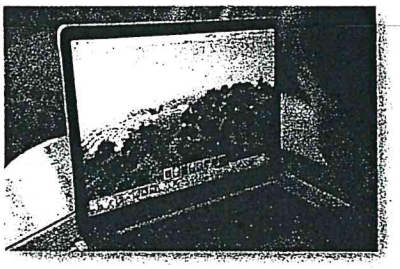
2. Octahedral dice have eight faces. Make a chart showing the sums of the faces on two dice. Which sum occurs most frequently? Which sums occur least frequently?

3. The heart honour cards (10, J, Q, K, A) are removed from a standard deck. Three cards are randomly selected from the heart honour cards without replacement.
   a) Illustrate all the possible outcomes using a tree diagram.
   b) Highlight the path that indicates the run queen of hearts, king of hearts, ace of hearts).
   c) How many possible outcomes are there?

2.2 The Fundamental Counting Principle, pages 70–75

4. A home security code requires five digits to be entered on a keypad.
   a) How many distinct security codes are possible?
   b) Sarah reset her security code but has forgotten it. If it takes her eight seconds per attempt, what is the maximum time it would take for her to find the correct code?

5. When ordering a gaming computer online, Ryan has three choices for processors, four choices for size of RAM, five choices for the video card, three choices for the hard drive, and two choices for the sound card.



   a) How many choices does Ryan have when configuring his computer?
   b) If there were an additional choice for the video card, how would it affect the total number of choices? Explain the difference.

6. Barb knits socks for a charity supporting homeless and low income people. She likes to make striped socks and selects from six different colours. The top stripe can be any colour. The second stripe may not match the first colour. The bottom colour may not match the second, but may match the first. How many distinct pairs of socks could Barb make?

2.3 Permutations and Factorials, pages 76–81

7. How many ways are there for a company to assign three different jobs to three of its five employees?

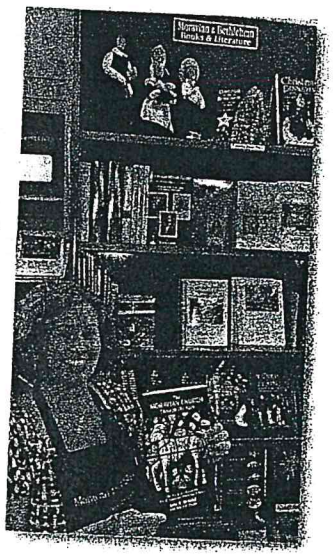
8. a) Evaluate each permutation and place them in the array as shown.

$$
\begin{array}{ccccccccc}
 & & & & {}_1P_1 & & & & \\
 & & & {}_2P_1 & & {}_2P_2 & & & \\
 & & {}_3P_1 & & {}_3P_2 & & {}_3P_3 & & \\
 & {}_4P_1 & & {}_4P_2 & & {}_4P_3 & & {}_4P_4 & \\
{}_5P_1 & & {}_5P_2 & & {}_5P_3 & & {}_5P_4 & & {}_5P_5
\end{array}
$$

   b) Extend the array by one row without using factorials. Explain how you did this.

   c) Identify and describe two other patterns in the array.

9. A bookstore clerk is arranging seven novels, four plays, and five poetry books in a display case. Each type of book remains in its own group, but the groups can be in any order. In how many ways could she arrange the books?



2.4 The Rule of Sum, pages 82–87

10. In how many ways could the letters in the word STORAGE be arranged if the vowels must remain in

   a) even positions?

   b) odd positions?

   c) even or odd positions?

11. In how many ways can you rearrange the letters in the word NATIVE if the vowels must not be together?

12. How many five-digit even numbers can be formed using all the digits 0, 1, 2, 3, and 4?

2.5 Probability Problems Using Permutations, pages 88–95

13. For a gift exchange, 10 people's names are written on slips of paper and placed in a bowl. The slips of paper are mixed up, and each person selects one name.

   a) What is the probability that everyone selects their own name?

   b) What is the probability that nobody selects his or her own name?

14. Six different coloured balls are placed in a box. Kendra and Abdul each select a ball without replacement.

   a) What is the probability that Kendra selects the green ball and Abdul selects the red ball?

   b) What is the probability that Kendra selects the green ball and Abdul does not select the red ball?

   c) What is the probability that Kendra does not select the green ball and Abdul does not select the red ball?

15. If five people each select a letter from the alphabet with repetition permitted, what is the probability that they are

   a) all the same?

   b) all different?

Chapter 2 Test Yourself

✓ Achievement Chart

Category	Knowledge/ Understanding	Thinking	Communication	Application
Questions	1, 2, 3, 4, 7, 8	11, 13, 14	4, 14	5, 6, 9, 10, 12

Multiple Choice

Choose the best answer for #1 to #3.

1. How many orders of faces are possible when a standard die is rolled four times?
   - A 16
   - B 24
   - C 1296
   - D 4096

2. Which of the following is equivalent to ${}_{101}P_{98}$?
   - A 3!
   - B $101 \times 100 \times 99 \times 98$
   - C $\frac{101!}{98!}$
   - D $\frac{101!}{3!}$

3. When flipping a coin five times, what is the probability that heads turns up every time?
   - A $\frac{1}{32}$
   - B $\frac{5}{32}$
   - C $\frac{1}{10}$
   - D $\frac{1}{25}$

4. Which of the following is not defined? Explain your reasoning.
   - ${}_{12}P_8$
   - ${}_{9}P_{10}$
   - ${}_{7}P_{0}$
   - ${}_{100}P_{100}$

Short Answer

5. Rosa is getting dressed and has decided that her shirt, pants, and socks are not to be the same colour. She has red, green, black, and blue of each.
   a) Draw a tree diagram illustrating her choices.
   b) How many choices does she have if she starts with a red pair of pants?

6. A hockey team has four left wingers, three right wingers, four centres, three left defence, four right defence, and two goalies. To create a starting lineup, a coach needs one player in each position. In how many ways could the starting lineup be chosen?

7. How many ways are there to assign five different roles in a play to the 12 members of a drama club?

8. There are three Canadians in the finals at a ski competition. Assuming all eight competitors are equally likely to win, what is the probability that the three Canadians will win gold, silver, and bronze?

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Extended Response

9. a) How many arrangements are there of the letters in the word COMPUTER?
   b) How many of them begin with a consonant?

10. In how many ways could the 11 members of a soccer team line up if the captain and assistant captain must remain apart?

11. There are 25 men and 20 women who belong to a club. An executive panel consisting of a president, vice president, secretary, and treasurer is being chosen.
   a) In how many ways could the executive panel be chosen with no restrictions?
   b) In how many ways could the executive panel be chosen if it must include at least one woman and one man?
   c) In how many ways could the executive panel be chosen if the president and vice president must have different genders?

12. Four letters are randomly selected from the alphabet. What is the probability that they are A, B, C, and D, in that order,
   a) if repetition is permitted?
   b) if repetition is not permitted?

13. Ten people each randomly select a number between 1 and 20. What is the probability that at least two of them select the same number?

14. To determine who should be the first dealer in a card game, one card is dealt to each of five players. The player with the card of the highest denomination gets to deal first.
   a) How many different results are possible when dealing to the five players?
   b) In how many ways could all players receive cards of different denominations?
   c) What is the probability that four players receive cards of the same denomination?
   d) How would the solution to part c) change if players each chose a card from a full deck instead of being dealt one?

Chapter Problem

Password Encryption

Consider four passwords you use, for bank cards, websites, and so on.

a) What are the rules for each? What must be included? What may be included? What must not be included?

b) What is the probability of someone guessing each password on the first try?

c) At 90 000 codes per second, how long, on average, would it take for a good password cracking program to break each of your passwords?

d) Some passwords require at least one digit and at least one capital letter. Why?

e) Develop a set of guidelines to identify a good, poor, or average password. Back it up with examples and calculations.

1.6 REVIEW EXERCISE

1. Explain why it makes sense to define 0! as 1.
2. Explain why $P(n, n) = n!$
3. Evaluate each expression.
   (a) $P(5, 1)$ (b) $P(8, 0)$ (c) $(5-5)!$ (d) $P(3, 3)$
4. Draw a tree diagram to show the possible orders for playing three record albums.
5. Evaluate each expression.
   (a) $P(6, 4)$ (b) $P(8, 5)$
   (c) $3!P(4, 2)$ (d) $\frac{9!}{5!4!} \times \frac{5 \times 4!}{3!2!}$
6. If the call letters of any Canadian television station must begin with the letter C, how many different stations could be named using three letters? What if four letters are allowed?
7. A series of history books has volumes A to G. In how many ways can these be arranged on a bookshelf so that they are *not* in order?
8. How many arrangements are there of the letters for each word given?
   (a) FRIENDS (b) REHEARSAL (c) BELLEVILLE
9. When Julia's parents return from Jamaica, they will give her a bracelet made of eight different sea shells with no visible clasp. How many different bracelets could have been made with these same shells? Why is the information about the clasp relevant?
10. In how many ways can seven people line up, single file, at the Capitol Theatre box office to purchase tickets for the Chinese Acrobats Show?
11. There are five speakers scheduled for a seminar on careers. How many different orders of speaking are possible if
    (a) there are no special conditions?
    (b) the marine biologist must speak first so that she can get back to her lab?
    (c) the electrical engineer and machinist are to speak one after the other?
    (d) the lawyer and paralegal are not to follow one another in the program?
12. Prove each statement for natural numbers n and r.
    (a) $P(n, 2) + P(n+1, 2) = 2n^2$.
    (b) $P(n, r) = P(n, 2)P(n-2, r-2)$
13. Solve the expression for $n \in N$.

$$P(2n+4, 3) = \frac{2}{3}P(n+4, 4)$$

1.7 CHAPTER 1 TEST

1. Evaluate the expression.

$$3! - 2 \times 4! + P(7,0) - P(3,3) + 0!$$

2. Students can travel to and from school by way of the park, the library, or the variety store. For example, they can go to school by way of the park and return by way of the library. Draw a tree diagram to show the possible routes taken to and from school on one day.

3. How many three-letter "words" can be formed from the letters of the word SHORTEN with no letter repeated?

4. In how many ways can the eight members of the board of directors of Pride International Corp. be seated around the round table in the board room?

5. At the opening assembly of the school year, the music students are to perform. The Senior Band knows seven pieces but the Junior Band knows only two so far, and the students of the Vocal Music class know three songs. Each group is to perform one piece.

(a) If the Junior Band must play first, followed by the Vocal Music students, and then the Senior Band, how many different programs are possible?

(b) If the groups may play in any order, how many possible programs would there be?

6. How many different arrangements of the word SUCCESS are there if

(a) there are no restrictions?

(b) the two vowels must be together?

7. How many odd four-digit numbers, all of the digits different, can be formed from the digits 0 to 7, if there must be a 4 in the number?

8. Five boats of various types are to be docked in the five slips at a small marina.

(a) In how many ways can they be docked so that the powerboat is at the end nearest the boathouse?

(b) In how many ways can they be located so that the two sailboats are at the extreme ends of the marina?

(c) In how many ways can they be located so that *either* the powerboat is nearest the boathouse *or* the sailboats are at the extreme ends?