

# Momentum-Impulse Theorem

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. A net force of 6.0 N acts on a 2.0 kg mass for 4.0 s. If the initial velocity of the mass is 3.0 m/s:

- a) What is the impulse on the mass?
- b) What is the final momentum of the mass?
- c) What is the final velocity of the mass?

$$\vec{I} = \Delta \vec{p} = \vec{F} \Delta t$$

a)  $\vec{I} = \vec{F} \Delta t = (6\text{ N})(4\text{ s}) = \boxed{24\text{ N}\cdot\text{s}}$  (or 24 kg·m/s)

b)  $\Delta \vec{p} = m\vec{v}_2 - m\vec{v}_1 = \vec{I}$   
 $m\vec{v}_2 = \vec{I} + m\vec{v}_1$   
 $= (24) + (2)(3)$   
 $\vec{p}_2 = m\vec{v}_2 = \boxed{30\text{ kg}\cdot\text{m/s}}$

using:  $\vec{F} = m\vec{a}$   
 $6 = 2\vec{a}$   
 $\vec{a} = 3\text{ m/s}^2$   
 $\vec{v}_2 = \vec{v}_1 + \vec{a} \Delta t$   
 $= (3) + (3)(4)$   
 $= 15\text{ m/s} \checkmark$

c)  $m\vec{v}_2 = 30\text{ kg}\cdot\text{m/s}$   
 $\vec{v}_2 = \frac{30}{2} = \boxed{15\text{ m/s}}$

2. A 5 kg object is moving with an initial speed of 20 m/s. It is acted upon by a constant net force for 6 seconds which results in the object slowing to a speed of 13 m/s.

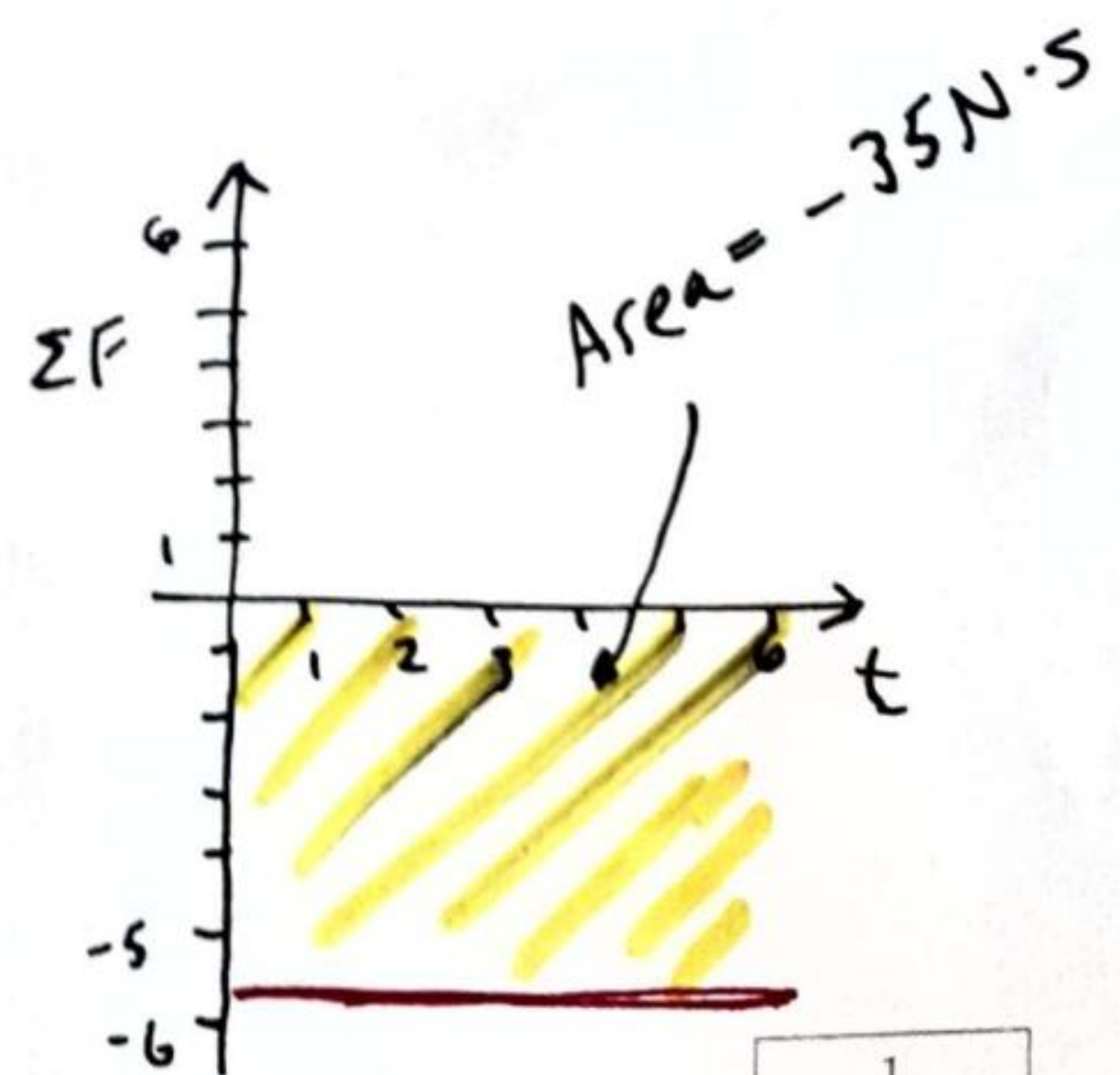
- a) Calculate the impulse on the object.
- b) Calculate the average net force acting on the object.

$\vec{v}_1 = 20\text{ m/s}$   
 $m = 5\text{ kg}$   
 $\Delta t = 6\text{ s}$   
 $\vec{v}_2 = 13\text{ m/s}$

a)  $\vec{I} = \Delta \vec{p} = m\vec{v}_2 - m\vec{v}_1$   
 $= m(\vec{v}_2 - \vec{v}_1)$   
 $= (5)(13 - 20)$   
 $\vec{I} = \boxed{-35\text{ N}\cdot\text{s}}$

negative impulse since the  $\Delta \vec{p}$  is negative.

b)  $\Sigma F_{\text{net}} = \vec{F}_{\text{net}} \Delta t = \vec{I}$   
 $\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{-35\text{ N}\cdot\text{s}}{6}$   
 $\boxed{\vec{F}_{\text{net}} = -5.8\text{ N}}$





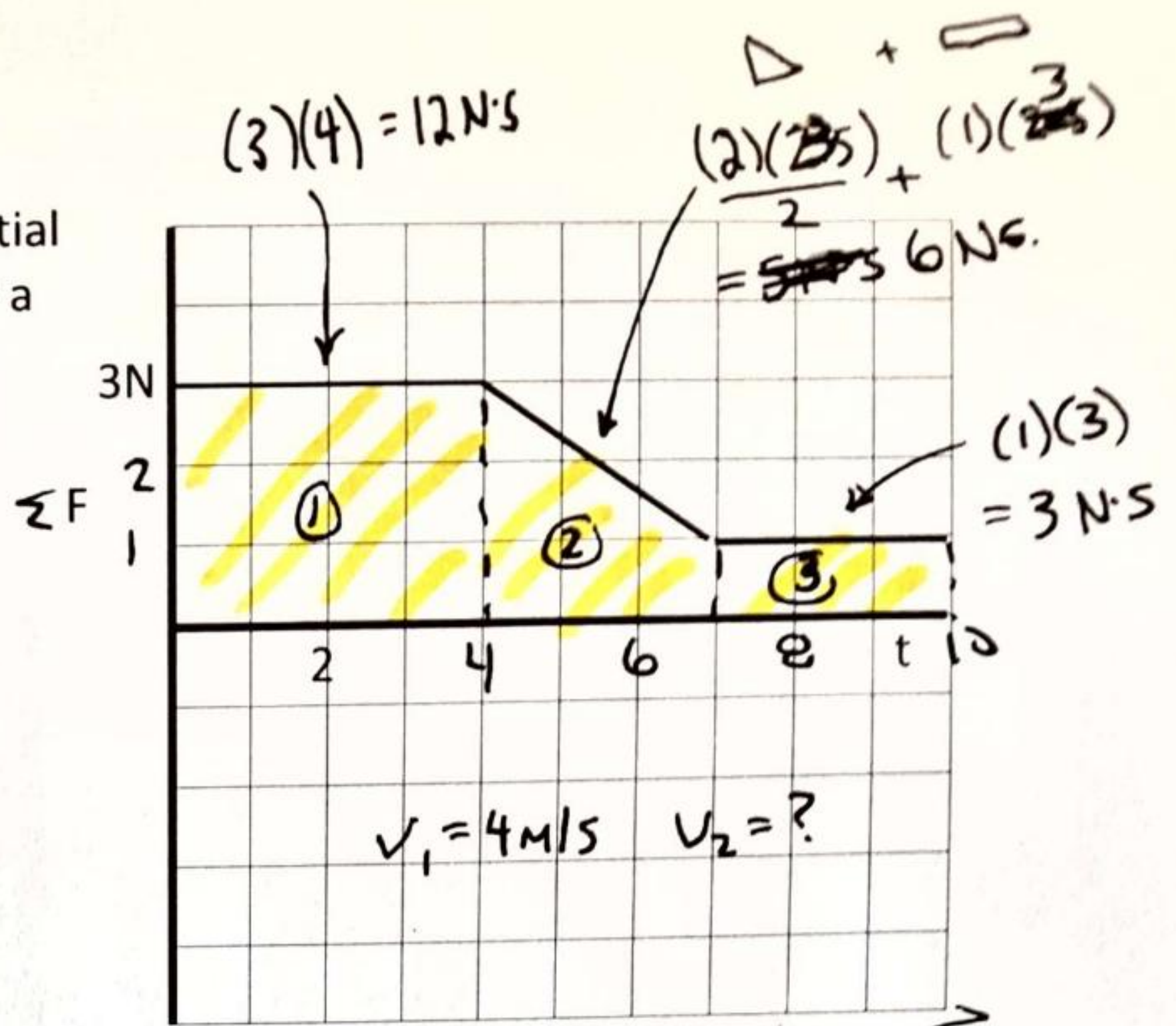
3. An object of mass, 5.0 kg, and an initial velocity of 4.0 m/s, is acted upon by a force shown in the graph.

- a) Determine the final velocity.
- b) Determine the average force.

a)  $\vec{I} = \text{AREA}$   
 $= \textcircled{1} + \textcircled{2} + \textcircled{3}$   
 $= 12 + 6 + 3$   
 $\vec{I} = 21 \text{ N}\cdot\text{s} \checkmark$

$\vec{I} = \Delta\vec{p} = m\vec{u}_2 - m\vec{u}_1$   
 $\Delta\vec{p} + m\vec{u}_1 = m\vec{u}_2$

$\vec{u}_2 = \frac{\Delta\vec{p} + m\vec{u}_1}{m} = \frac{(21) + (5)(4)}{5} = 8.2 \text{ m/s} \checkmark$



b)  $\sum F_{\text{avg}} \Delta t = \vec{I}$   
 $\sum \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} = \frac{21 \text{ N}\cdot\text{s}}{10 \text{ s}}$   
 $\sum F_{\text{avg}} = 2.1 \text{ N} \checkmark$

4. An object of mass, 4.0 kg, and initial velocity, 10 m/s, is acted upon by a force as shown in the graph

- a) Determine the final velocity.
- b) Determine the average force.

a)  $\vec{u}_1 = 10 \text{ m/s} \quad m = 4 \text{ kg}$

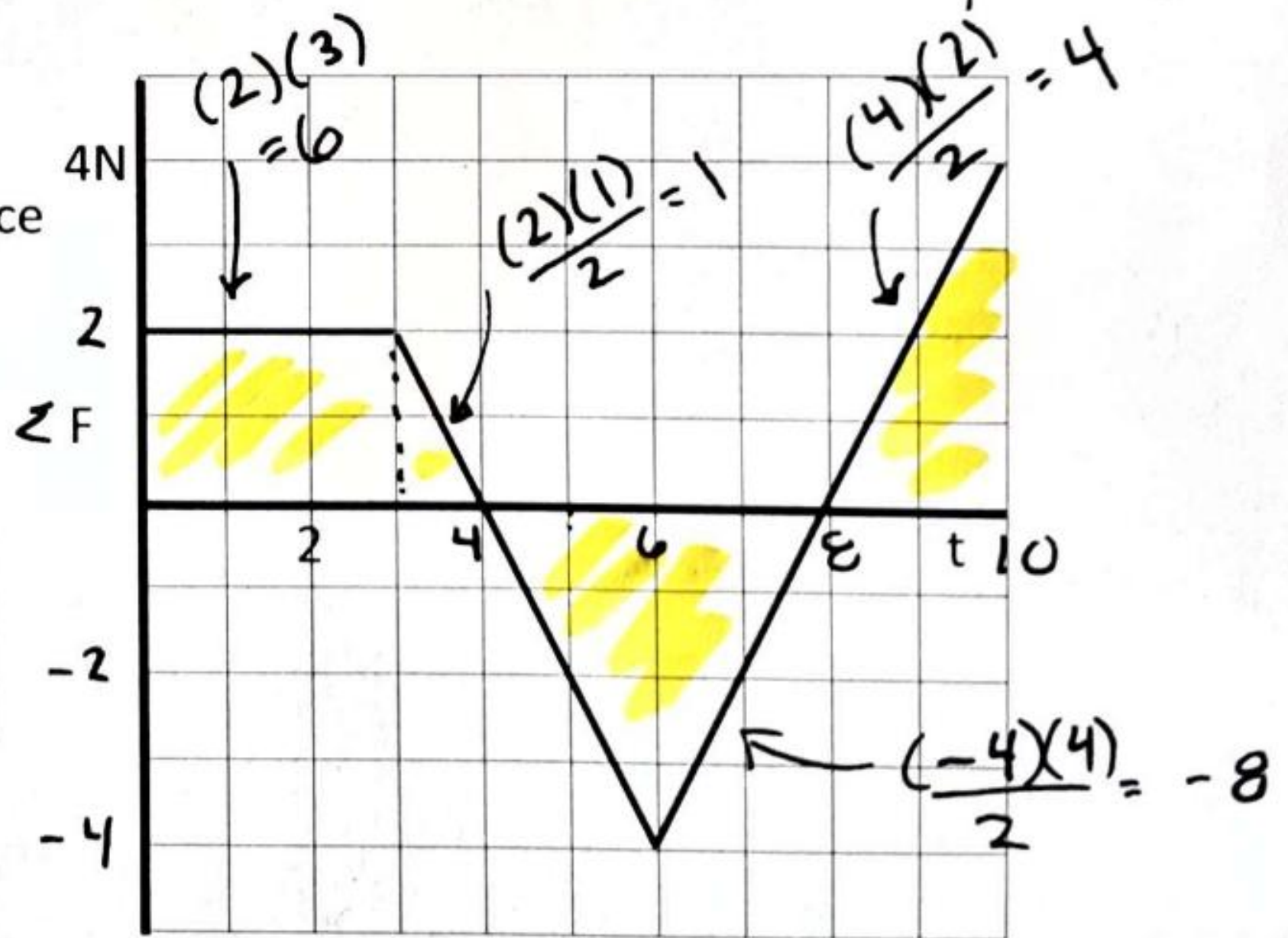
From #3

$\vec{u}_2 = \frac{\Delta\vec{p} + m\vec{u}_1}{m}$

where  $\Delta\vec{p} = \vec{I} = \text{AREA}$ .

$\vec{I} = \Delta\vec{p} = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4}$   
 $= 6 + 1 + 4 - 8$   
 $\vec{I} = 3 \text{ N}\cdot\text{s}$

$\therefore \vec{u}_2 = \frac{(3) + (4)(10)}{4} = 10.75 \text{ m/s} \checkmark$



b)  $\sum \vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t}$   
 $= \frac{3 \text{ N}\cdot\text{s}}{10 \text{ s}}$

$\sum F = 0.3 \text{ N} \checkmark$



5. The average accelerating force exerted on a 5.0 kg shell in a gun barrel is  $5.0 \times 10^4$  N, and the muzzle velocity is 200 m/s. Calculate (a) the impulse on the shell, and (b) the length of time it takes to move up the heavy gun barrel.

[ Ans:  $1.0 \times 10^3$  Ns;  $2.0 \times 10^{-2}$  s ]

$$\sum \vec{F}_w = 5 \times 10^4 \text{ N}$$

$$v_1 = \emptyset$$

$$v_2 = 200 \text{ m/s}$$

$$m = 5 \text{ kg.}$$

$$\begin{aligned} \text{a) } \vec{I} &= \Delta \vec{p} \\ &= m\vec{v}_2 - m\vec{v}_1 \\ &= m(\vec{v}_2 - m\vec{v}_1) \\ &= 5(200 - \emptyset) \end{aligned}$$

$$\boxed{\vec{I} = 1000 \text{ N}\cdot\text{s}} \checkmark$$

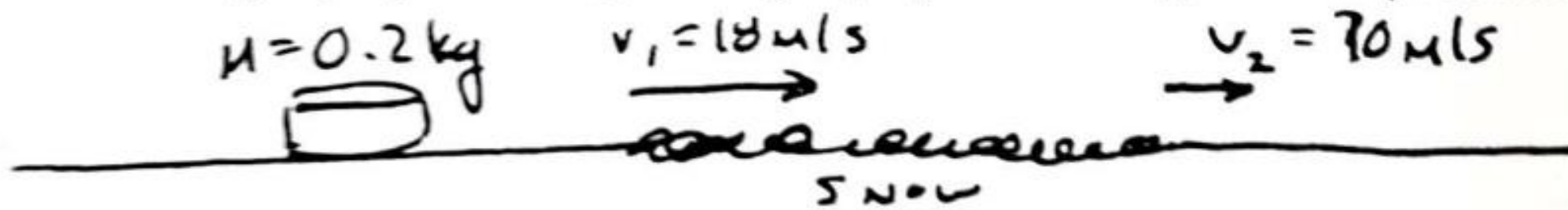
$$\text{b) } \sum \vec{F}_w = \frac{\vec{I}}{\Delta t} \Rightarrow \Delta t = \frac{\vec{I}}{\sum \vec{F}_w} = \frac{1000}{5 \times 10^4}$$

$$\boxed{\Delta t = 0.02 \text{ s}}$$

6. A hockey puck of mass 0.20 kg is sliding along a smooth, flat section of ice at 18 m/s when it encounters some snow. After 2.5 s of sliding through the snow, it returns to smooth ice, continuing to slide at a speed of 10 m/s.

- a) What is the change in momentum of the puck?  
 b) What impulse does the snow exert on the puck?  
 c) What average frictional force does the snow exert on the puck?

[ Ans: -1.6 kg m/s (forward), 1.6 kg m/s (backward), 0.64 N (backward) ]



$$\text{a) } \left. \begin{aligned} \vec{I} &= \Delta \vec{p} = \sum \vec{F}_w \Delta t \\ \Delta \vec{p} &= m\vec{v}_2 - m\vec{v}_1 \\ &= (0.2)(10 - 18) \end{aligned} \right\} \boxed{\Delta \vec{p} = -1.6 \text{ N}\cdot\text{s} \text{ or } -1.6 \text{ kg}\cdot\text{m/s}}$$

$$\text{b) } \vec{I} = \Delta \vec{p} = -1.6 \text{ kg}\cdot\text{m/s} \checkmark$$

$$\text{c) } \sum \vec{F}_w \Delta t = \vec{I}$$

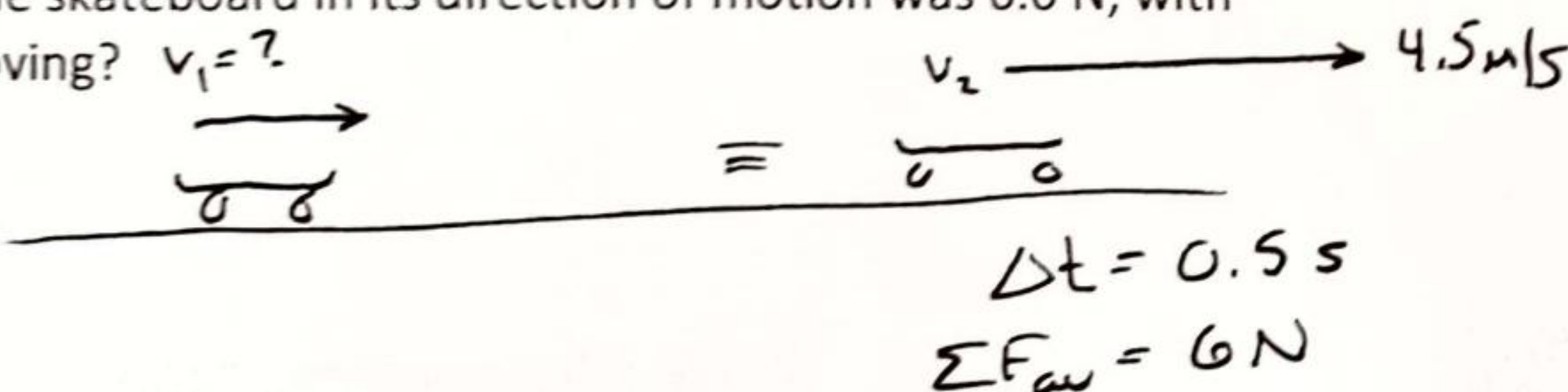
$$\sum \vec{F}_w = \frac{\vec{I}}{\Delta t} = \frac{-1.6 \text{ N}\cdot\text{s}}{2.5 \text{ s}} = -0.64 \text{ N [forward]} \text{ or } +0.64 \text{ N [backward?]}$$

(using Newton's 2<sup>nd</sup> law (same as impulse-momentum theorem!))  $a = \frac{\sum F}{m} = \frac{v_2 - v_1}{\Delta t} \times m$

$$\sum F = -0.64 \text{ N} \checkmark \quad 3$$



7. A 2.0 kg skateboard is rolling across a smooth, flat floor when a small girl kicks it, causing it to speed up to 4.5 m/s in 0.50 s without changing direction. If the average force exerted by the girl on the skateboard in its direction of motion was 6.0 N, with what initial velocity was it moving?  $v_1 = ?$   
 [ Ans: 3.0 m/s (forward) ]



$$\vec{I} = \Delta \vec{p} = \vec{F} \Delta t$$

$$= (6)(0.5)$$

$$\Delta \vec{p} = 3 \text{ N}\cdot\text{s}$$

$$M_2 \vec{u}_2 - M_2 \vec{u}_1 = 3 \text{ N}\cdot\text{s}$$

$$M_2 \vec{u}_2 = 3 + M_2 \vec{u}_1$$

$$M \vec{u}_2 - 3 = M_2 \vec{u}_1$$

$$\frac{M \vec{u}_2 - 3}{M} = \vec{u}_1$$

$$\vec{u}_1 = \frac{(2)(4.5) - 3}{(2)}$$

$$\vec{u}_1 = 3 \text{ m/s [forward]}$$

since +

8. A 200 kg shot is discharged horizontally from a cannon, of mass  $2.0 \times 10^4$  kg, with a speed of 250 m/s relative to the ground.  
 a) Find the steady force which, acting on the cannon will stop its recoil in 2.0 s.  
 b) How far will the cannon recoil?  
 [ Ans:  $2.5 \times 10^4$  N; 2.5 m ]

a)  $v_1 = 0$   $v_2 = 250 \text{ m/s}$   
 $m = 200 \text{ kg}$

$$\Sigma \vec{F}_{av} = \frac{\Delta \vec{p}}{\Delta t}$$

$$= \frac{M \vec{u}_2 - M \vec{u}_1}{\Delta t}$$

$$= \frac{(200)(250 - 0)}{(2)}$$

$$\Sigma \vec{F}_{av} = 2.5 \times 10^4 \text{ N}$$

\* Newton's 3<sup>rd</sup> law: the force on the cannon is the same as the force on shot !!!

b)  $\Delta d = ?$  use kinematics and  $F = ma$

$$a = \frac{\Sigma \vec{F}}{M} = \frac{2.5 \times 10^4 \text{ N}}{2 \times 10^4} = 1.25 \text{ m/s}^2$$

cannon!

$$\Delta d = v \Delta t + \frac{a \Delta t^2}{2}$$

$$\Delta d = \frac{(1.25)(2)^2}{2} = 2.5 \text{ m}$$