12. a) 45 configurations
   b) Adding a siding colour results in an increase of 15 choices. An additional trim colour results in only 9 more choices.

13. a) 2 b) 19 c) 4

Extend

14. There are 1 190 000 different local phone numbers Sarah can call.
15. Assume rows on checkerboard are numbered 0 to 7. The portion of the tree diagram that starts with a move diagonally left has 49 possible paths to the opposite side. Similarly, the portion of the tree diagram that starts with a move diagonally right has 54 possible paths to the opposite side. The total number of possible paths to the opposite side is 49 + 54, or 103.
16. a) 8 b) 16 c) 2^n

2.2 The Fundamental Counting Principle, pages 70–75

Example 1 Your Turn
60

Example 2 Your Turn
a) 308 915 776 b) 19 770 609 664

Example 3 Your Turn
13 800

Reflect

R1. Johnny is wrong. He should apply the fundamental counting principle: $4 \times 8 \times 3 = 96$.
R2. Answers may vary. The fundamental counting principle is the product of the number of ways multiple events can occur. For example, there are 3 flavours of ice cream and 6 choices of toppings to create a sundae. Event one, choose ice cream flavour, can happen in 3 ways. Event two, choosing a topping, can happen in 6 ways. The result in 3×6, or 18 different 1-topping ice cream sundaes.

Practise

1. a) 4 b) 8 c) 16 d) 2^n
2. a) 210 b) 2730
3. 240
4. a) 3 b) appetizers: 4, main course: 5, dessert: 3
   c) 60
5. C
6. B
7. a) 25 b) 20

Apply

8. a) 16 b) 64 c) 64 d) 4096
   e) 144 f) 248 832 g) n^k
9. 19 000
10. 90
11. 7776
12. a) 12 960 000 b) 11 703 240
13. a) 216 000 b) 205 320
15. a) 456 976 000 b) 17 576 000 c) 1 000 000
16. An Alberta licence plate will have much fewer choices than an Ontario licence plate. There are 26 choices for a letter, while there are only 10 choices for a digit.
17. 24
18. The same. In event one and two, the colour of the dice does not affect the choices for a die, and rolling three dice once has the same results as rolling one die three times.
19. Answers may vary.
   a) My security code is 325. I pressed ENTER 145 times before I saw my code. The actual number of possible outcomes for a three-digit security is 1000.
   b) It might take 100 times longer to break a five-digit code versus a three-digit code. The actual number of possible outcomes for a five-digit security is 100 000. Using a graphing calculator to randomly generate a five-digit code could possibly take more than 10 000 presses of ENTER because of duplicates or your code may never be generated.
20. Answers may vary. To find the number of choices for each of the three toppings, factor 4080: $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 17$. Using all the factors, create three values. For example, there could be $(2 \times 2 \times 2)$ choices for sauce, $(2 \times 3 \times 5)$ choices for actual topping ingredient, and 17 choices for cheese. Another possibility is that this includes four different sizes (S, M, L, XL). Then, there are actually 1020 $(2 \times 2 \times 3 \times 5 \times 17)$ topping options.
21. a) 1 048 576 b) 9 756 625

Extend

22. 52 400 215 plates
23. a) 60 b) 52
24. 8223
25. 1 275 120

2.3 Permutations and Factorials, pages 76–81

Example 1 Your Turn
a) 24 b) 720
c) 7920 d) 144

Example 2 Your Turn
40 320

Example 3 Your Turn
59 280

Example 4 Your Turn
240

Reflect

R1. Arranging r people from a group of n people with regard to order will have more possibilities. For example, ABC can be arranged in 6 ways: ABC, ACB, BAC, BCA, CAB, CBA. Without regard for order these arrangements are the same.
R2. Answers may vary. Using a calculator, 0! has a value of 1. Look at the formula for permutations. Suppose three people are awarded first, second, and third prize. So, $r = n = 3$ and the formula becomes $\frac{3!}{0!}$. In order for this to make sense, choose 0! to be defined as 1.

Practise

1. a) 362 880 b) 3 991 680 c) 5040 d) 6720
2. a) $\frac{6!}{2!}$ b) $\frac{15!}{9!}$ c) 7! d) $\frac{8!}{4!}$
   e) $\frac{n!}{(n-4)!}$ f) $(n + 1)!$
3. a) ${}_6P_6$ b) ${}_{91}P_6$ c) ${}_{18}P_6$
4. C 5. B 6. 2730 7. 73 440

Apply

8. ${}_{22}P_9$, or 180 503 769 600
9. a) 10! b) 99! c) 10!
   d) $n!$ e) $(n + 2)!$

10. a) 15!, or 1 307 674 368 000 b) 32 760
c) 13 650
11. a) 3 628 800 b) 604 800
12. 3 720 087 14. 3 628 800 15. 207 3660 000

Extend
16. a) $n = 11$ b) $n = 6$
17. 371 589 120
18. 23!, or 25 852 016 738 884 976 640 000
19. a) $\frac{9!}{8!!}$ b) $\frac{(2k+1)!}{2k!!}$ c) $2^n n!$
20. 6

2.4 The Rule of Sum, pages 82–87

Example 1 Your Turn
a) 408 240
b) 46 080

Example 2 Your Turn
a) ${}_{13}P_3$
b) ${}_{4}P_3$
c) 1739

Example 3 Your Turn
480

Reflect
R1. It is simpler to calculate the number of executives without any males or females than all the possibilities for at least one male and one female.
R2. Use the fundamental counting principle when the events are independent. For example, rolling a die twice. The outcome of the first event does not affect the second. Use the rule of sum when events are mutually exclusive. For example, rolling a 1 or a 2. Both events cannot happen at the same time.

Practise
1. 2016
2. a) 8 b) 18
3. A 4. D

Apply
5. a) 130 b) 78
6. 182 520 000 7. 173 659 200
8. a) 120 100 b) 980 200
c) When the answers in parts a) and b) are expanded into factorial form, all three expressions in part b) are at least 2 times as big as those in part a). So, the result is more than $2^3 = 8$ times the answer in part a).
9. a) 60
b) Answers may vary. Question: Five speakers, P, Q, R, S, and T, are available to address a meeting. The organizer must decide whether to have four or five speakers. How many options would the organizer have for the meeting? Answer: $5! + 4! = 144$. There are 144 options.
10. 61 328 11. 3 628 799
12. a) 48 b) 126
13. Answers may vary. For each roll of two dice, there are six ways to get doubles. There are 6 + 6 + 6, or 18 ways to get doubles in one or two or three rolls.
14. Morse code is used to represent 26 letters, 10 digits, and 8 punctuation symbols, or a total of 44 symbols. Since each character has two options (dot or dash), a maximum of six characters is needed: $2^6 = 64$.

Extend
16. 82
17. a) 9 b) 44
18. a) 265 b) 455 c) 1

2.5 Probability Problems Using Permutations, pages 88–95

Example 1 Your Turn
a) $P(\text{all same}) = \frac{1}{10\,000\,000\,000}$
b) $P(\text{all 6s}) = \frac{1}{7776}$
For independent trials,
$P(\text{all the same}) = (P(\text{a success}))^{\#\text{ trials}}$.

Example 2 Your Turn
$P(\text{in grade order}) = \frac{1}{24}$

Example 3 Your Turn
a) $P(\text{ace, ace, ace, jack, jack}) = \frac{1}{1\,082\,900}$
b) $P(\text{heart, heart, club, club, club}) = \frac{143}{166\,600}$

Example 4 Your Turn
a) approximately 0.7164 b) approximately 0.2836

Reflect
R1. No. The probability that at least two people have the same birthday is approximately 0.6269.
R2. Answers may vary. If the trials are dependent, permutations can be used. Look for restrictions such as, "without replacement" or "alphabetical order."
R3. Answers may vary. The first represents 3 of 12 objects being arranged. The second is 3 times 1 of 12 objects being arranged.

Practise
1. $P(\text{king, queen, jack}) = \frac{8}{16\,575}$
2. $\frac{1}{15}$ 3. A 4. C

Apply
5. $\frac{1\,307\,674\,367\,999}{1\,307\,674\,368\,000} : \frac{1}{1\,307\,674\,368\,000}$
6. a) approximately 0.000 505
b) $\frac{{}_{30}P_3}{{}_{365}P_3} \approx 0.000\,505$
7. a) $P(\text{doubles}) = \frac{1}{6}$ b) $P(\text{doubles twice}) = \frac{1}{36}$
c) They are the same.
8. a) $P(\text{3 boys}) = \frac{1}{8}$ b) $P(\text{4 boys}) = \frac{1}{16}$
c) $P(\text{5 boys}) = \frac{1}{32}$ d) $P(n\text{ boys}) = \frac{1}{2^n}$
9. a) $P(\text{MATH}) = \frac{1}{3024}$ b) $P(\text{M,A,T,H}) = \frac{1}{126}$
c) $P = \frac{4}{9}$
10. a) $P(\text{ascending order}) \approx 4.1697 \times 10^{-5}$
b) $P(\text{no same denomination}) \approx 0.2102$
11. $P(\text{at least two the same}) \approx 0.4114$
12. 23
13. a) $P(\text{songs in order}) = \frac{1}{3\,628\,800}$
b) $P = \frac{1}{45}$
14. a) 0.8203:0.1797 b) 0.4160:0.5840
15. Answers may vary. Example: 7

16. a) i) $\frac{1}{38\,955\,840}$ ii) $\frac{1}{78\,960\,960}$ iii) $\frac{1}{146\,611\,080}$
b) The probability of cracking the safe decreases as the five different numbers are chosen from a greater range of number.
18. The probability that at least two people have the same birthday as you is approximately 0.5687.
19. a) not throwing a sum of 7 on consecutive rolls
b) three different letters being arranged in alphabetical order
c) two out of five friends having the same birth month

Extend

20. 3.1664×10^{-7}
21. Answers may vary. Any scenario that has $n(A) = 1$ and $n(S) = {}_{15}P_7$. For example, winning first prize similar to question 20.
22. a) approximately 0.0947
b) approximately 6.9613×10^{-5}
23. a) approximately 0.0188
b) approximately 0.1004
24. a) approximately 2.2355×10^{-6}
b) approximately 0.0026

Chapter 2 Review, pages 96–97

1. 27 possible outcomes
2.

Second Die \ First Die	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

The sum of 9 occurs eight times. There is only one occurrence of the sum 2 and sum 16.

3. a) 60 possible outcomes b) (Q, K, A) c) 60
4. a) 100 000 b) 800 000 s, or about 9.3 days
5. a) 360
b) Ryan has 432 choices to configure his computer. Increasing the number of choices for any option will increase the total number of possible configurations.
6. 150
7. 60
8. a) and b)

```
                1
             2     2
          3     6     6
       4    12    24    24
    5    20    60   120   120
 6    30   120   360   720   720
```

The first term in row n is n. To obtain the remaining terms in row n, multiply all the terms in the row above by n.

c) Answers may vary. The last term in row n equals $n!$. The last two terms in each row are equal.
9. 87 091 200
10. a) 144 b) 576 c) 5040
11. 576
12. 60
13. a) approximately 2.7557×10^{-7}
b) $1 - 2.7557 \times 10^{-7}$
14. a) $\frac{1}{30}$ b) $\frac{2}{15}$ c) $\frac{29}{30}$
15. a) approximately 8.4165×10^{-8}
b) $1 - 8.4165 \times 10^{-8}$

Chapter 2 Test Yourself, pages 98–99

1. C 2. D 3. A
4. ${}_9P_{10}$ is not defined, $n < r$.

$${}_9P_{10} = \frac{9!}{(9-10)!} = \frac{9!}{(-1)!}$$

5. a) 24 possible outcomes b) 6
6. 1152 7. 95 040 8. $\frac{1}{56}$
9. a) 40 320 b) 25 200
10. 32 659 200
11. a) 3 575 880 b) 3 156 000 c) 1 806 000
12. a) $\frac{1}{456\,976}$ b) $\frac{1}{358\,800}$
13. approximately 0.9345
14. a) 311 875 200 b) 158 146 560
c) approximately 3.6938×10^{-6}
d) approximately 3.5013×10^{-5}

Chapter 3 Combinations

Prerequisite Skills, pages 102–103

1. a) 40 320 b) 60 480 c) 144 d) 151 200
e) 1320 f) 35 g) 330 h) 504 504
2. a) $n!$ is a product of sequential natural numbers with the form $n! = n(n-1)(n-2) \times \ldots \times 2 \times 1$.
b) The number of permutations of r items from a collection of n items is written as ${}_nP_r$ or $P(n, r)$. ${}_nP_r = \frac{n!}{(n-r)!}, n \geq r$
3. a) $\frac{7!}{4!}$ b) $\frac{100!}{8!}$ c) $\frac{n!}{(n-6)!}$ d) $\frac{15!}{(15-r)!}$
4. a) 40 320 b) 6720 c) 1716
5. a) 39 916 800 b) 86 400
6. a) 40 320 b) 336
7. a) The first and last terms are 1. The remaining terms are the sum of the two adjacent terms in the row above.

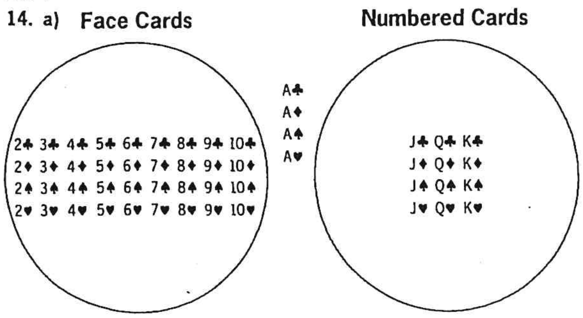
```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5  10  10   5   1
1   6  15  20  15   6   1
```

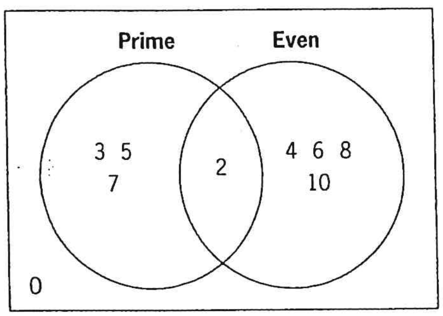
b) Answers may vary. Consider the top of the triangle row 0. Then, the sum of entries in row n equals 2^n. The second diagonal contains the counting numbers 1, 2, 3, 4, 5, ...
8. a) $\frac{1}{8}$ b) $\frac{1}{8}$ c) $\frac{1}{8}$ d) $\frac{3}{8}$
9. a) approximately 0.0060 b) approximately 0.2549
c) approximately 0.3077
10. a) $\frac{1}{18}$ b) $\frac{1}{18}$ c) $\frac{1}{9}$
d) approximately 1.5619×10^{-16}

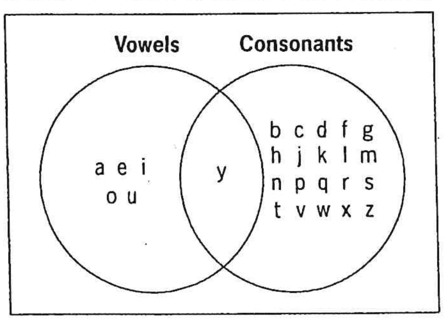
11. a) It could have six faces, two of each colour.
b) 18 c) $\frac{1}{18}$ d) $\frac{2}{3}$

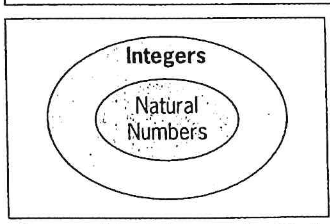
12. The events A and B are not mutually exclusive, since the overlap shows there are common elements. If A and B are non-mutually exclusive events, then the total number of favourable outcomes is:
$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.

13. 7

14. a)


b)


c)


d)


15. a) x^6 b) $4a^2$ c) $25m^6$ d) $81k^{12}$

16. a) $x^2 + 2xy + y^2$ b) $a^3 + 3a^2b + 3ab^2 + b^3$
c) $4p^2 + 4pq + q^2$

17. a) $(n-1)(n-2)$ b) $n(n-1)$ c) n

3.1 Permutations With Non-Ordered Elements, pages 104–109

Example 1 Your Turn

a) In each case, there are 3! permutations.
b) In each case, there are 2!2! permutations.

Example 2 Your Turn

I would expect the number of orders of the second team to be higher.

Example 3 Your Turn

840

Reflect

R1. There are four identical 2s.

R2. No. The number of permutations of three girls and four boys, or seven people, is 7!
The number of permutations of three red balls and four green balls is $\frac{7!}{3!4!}$. All red balls are identical and all green balls are identical.

R3. Answers may vary. It is much quicker to use the formula. Drawing a tree diagram or chart may not be practical and takes longer.

Practise

1. a) 2520 b) 1680
c) 420 d) 1 905 780 240

2. B

3. D

4. a) 20 160 b) 420 c) 415 800 d) 180

5. a) 60 b) 20 c) 30 d) 5

Apply

6. a) 369 600 b) 34 650 c) 924

7. 70 8. 2520 9. 9 459 450

10. Since it is not possible to have 0 or a fractional number of ways to do something, the number of permutations involving identical objects will always be a natural number. The denominator must be a factor of the numerator.

11. 462; Assume that the streets are laid out in a grid pattern, and that all of the streets are continuous between his school and his home.

12. a) 369 600 b) 7 484 400

13. 42

14. 24

16. a) 10 764 000 b) 43 056 000 c) 1 794 000
d) The number of licence plates in part c) is about 0.4% of the total licence plates without restrictions.

17. Answers may vary. How many arrangements are there of 12 flags in a row if two are red, three are green, four are blue, and three are yellow?

Extend

18. 1320 19. 60 20. 185 794 560

3.2 Combinations, pages 110–115

Example 1 Your Turn

a) 210 b) 252 c) 210

Example 2 Your Turn

525

Example 3 Your Turn

35

Reflect

R1. Answers may vary.
a) For permutations, order matters. For example, select five out of eight for five different offices of the committee.
b) For combinations, order does not matter. For example, select five out of eight for a committee.

R2. Answers may vary. Examples: selecting groceries, selecting toppings for a sandwich

R3. A situation in which order matters (permutations) will have more possibilities. ${}_nC_r = \frac{{}_nP_r}{r!}$

For each combination of r items there are $r!$ permutations. So, the number of combinations is $r!$ times smaller than the number of permutations.

Practise

1. a) 126 b) 70 c) 220 d) 462 e) 420 f) 27 772 222 500
2. B
3. B
4. 210
5. 330
6. 1
7. a) 1 b) 1 c) 1 d) 1 e) 1

Apply

8. 168
9. a) 65 780 b) 792 c) 575 757 d) 845 000 e) 1 096 680
10. a) 5 586 853 480 b) 3 838 380
    c) There is a larger number of ways to choose a 12-person jury than a 6-person jury. The denominator in part a) (28!12!) is smaller than that in part b) (34!6!).
11. a) 330 b) 150 c) 60 d) 5 e) 15
    f) Parts b) to e) are subsets of part a). The combinations add to 230. With the inclusion of a three truck and one car option, the total is 330.
12. a) This is a combination situation, since the order does not matter.
    b) ${}_{14}C_2 = 91$
13. a) 210 b) 252
    c) ${}_{10}C_n$, $3 \le n \le 10$
14. a) i) ${}_7C_2 = 21$, ${}_7C_5 = 21$
    ii) ${}_4C_3 = 4$, ${}_4C_1 = 4$
    iii) ${}_{12}C_4 = 495$, ${}_{12}C_8 = 495$
    The values in each pair are the same.
    b) ${}_nC_r = {}_nC_{n-r}$. The only difference is the order of the terms in the denominator. The number of combinations of n items taken r at a time is equivalent to the number of combinations of n items taken $n - r$ at a time.
    c) $$\begin{aligned}{}_nC_r &= \frac{n!}{(n-r)!r!}\\ &= \frac{n!}{r!(n-r)!}\\ &= \frac{n!}{(n-(n-r))!(n-r)!}\\ &= {}_nC_{n-r}\end{aligned}$$
15. a) 6 126 120 b) 6 126 120
    c) The results for parts a) and b) are the same. The order in which the jobs are assigned is irrelevant.
16. a) 20
    b) The general formula for the number of diagonals in a polygon with n sides is $\frac{n(n-3)}{2}$. Using combinations, select two points from the n vertices: ${}_nC_2$. However, this also includes consecutive vertices that form a side of the polygon. So, subtract n, the number of sides. There are ${}_nC_2 - n$ diagonals in an n-sided convex polygon.
18. 756 756
19. 2 375 880 867 360 000
20. The techniques from the two sections result in the same answer.
21. There are ${}_{30}C_5 \times {}_{25}C_5 \times {}_{20}C_5 \times {}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$ ways to divide a class of 30 students into six teams of five members.
    The number of ways to arrange a total of 30 balls with six different colours is also ${}_{30}C_5 \times {}_{25}C_5 \times {}_{20}C_5 \times {}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$.
22. For $r > 0$, there will always be more r-permutations of n items than r-combinations or n items.
    ${}_nC_r = \frac{{}_nP_r}{r!}$
    In permutations order matters, not in combinations. For each combination of r items there are $r!$ permutations. So, the number of combinations is $r!$ times smaller than the number of permutations.
23. 42

Extend

24. a) Let the three consecutive numbers be represented by n, $n - 1$, and $n - 2$.
    $$\frac{n(n-1)(n-2)}{3!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!3!} = {}_nC_3$$
    b) Let the r consecutive numbers be represented by $n, n-1, n-2, \dots, (n-r+1)$.
    $$\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} = \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!r!} = {}_nC_r$$
25. $n = 5$
26. 816

3.3 Problem Solving With Combinations, pages 116–121

Example 1 Your Turn

Combinations:

$${}_8C_1 + {}_8C_2 + {}_8C_3 + {}_8C_4 + {}_8C_5 + {}_8C_6 + {}_8C_7 + {}_8C_8$$
$$= \frac{8!}{(8-1)!1!} + \frac{8!}{(8-2)!2!} + \frac{8!}{(8-3)!3!} + \frac{8!}{(8-4)!4!} + \frac{8!}{(8-5)!5!} + \frac{8!}{(8-6)!6!} + \frac{8!}{(8-7)!7!} + \frac{8!}{(8-8)!8!}$$
$$= 255$$

Indirect Method:

$$2^8 - 1 = 256 - 1 = 255$$

Example 2 Your Turn

a) 576 050 767 488 b) 2 672 060 c) 4 306 559 400 d) 181 823 183 256 e) 158 362 127 352

Example 3 Your Turn

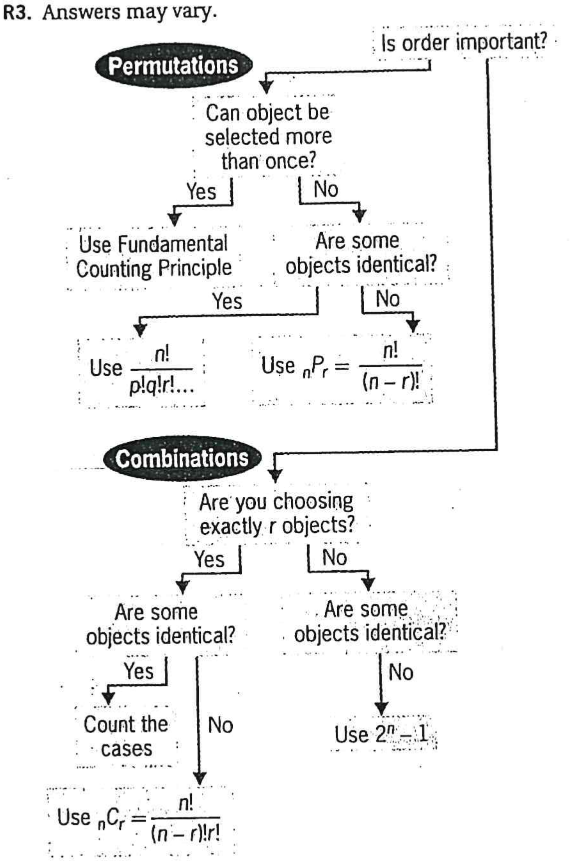
a) 2 052 000 b) 307 800

Reflect

R1. When determining the total number of subsets of a set, you add the number of possibilities in each case because the events are mutually exclusive.

R2. When using cases to determine the number of ways of selecting objects from different sets, you add because the events are mutually exclusive.

R3. Answers may vary.



Practise

1. 15 2. B 3. C
4. a) 6720 b) 5880 c) 2016 d) 14 826

Apply

5. a) combinations; The order in which the 5 members are chosen does not matter.
   b) permutations; Order matters, since each position holds an office.
   c) both; The order in which the members of the team are chosen does not matter. When arranging for a photo, order matters.
   d) permutations; Order matters because there are 3 different prizes.
6. 32 767 7. 92 8. 63 9. 15 10. 14 400
11. a) 1 237 792 b) 6 799 260 c) 3 219 112
12. a) 5 326 270 b) 6 864 396 000
13. 600 14. 2 041 200 000 15. 160

Extend

17. 968 18. 2 560 481 280

3.4 Combinations and Pascal's Triangle, pages 122–127

Example 1 Your Turn

1 + 7 + 28 + 84 + 210 = 330.
Comparing the terms in Pascal's triangle to combinations gives $_6C_6 + {}_7C_6 + {}_8C_6 + {}_9C_6 + {}_{10}C_6 = {}_{11}C_7$.

Example 2 Your Turn

Pascal's Method:

School

210	84	28	7		1
126	56	21	6		1
70	35	15	5		1
35	20	10	4		1
15	10	6	3		1
5	4	3	2		1
1	1	1	1	Home	

Bill can take 210 different routes to school.
Combinations: $_{10}C_4 \times {}_6C_6 = 210$

Example 3 Your Turn

a) $(a + b)^4 = {}_4C_0a^4b^0 + {}_4C_1a^3b^1 + {}_4C_2a^2b^2 + {}_4C_3a^1b^3 + {}_4C_4a^0b^4$
$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
The terms $_4C_r$, where $r = 0$ to 4 correspond to row 4 in Pascal's triangle. The degree of each term is 4.

b) $(p + q)^5 = {}_5C_0p^5q^0 + {}_5C_1p^4q^1 + {}_5C_2p^3q^2 + {}_5C_3p^2q^3 + {}_5C_4p^1q^4 + {}_5C_5p^0q^5$
$= p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$
The terms $_5C_r$, where $r = 0$ to 5 correspond to row 5 in Pascal's triangle. The degree of each term is 5.

Reflect

R1. Answers may vary. The term labels begin with $t_{0,0}$. This maintains the pattern of first and last terms in each row both being 1, since there is only one term.
R2. The terms in row n of Pascal's triangle correspond to the combinations $t_{n,r} = {}_nC_r$. Each row in Pascal's triangle represents the combinations of choosing 0 items, 1 item, 2 items, and so on, out of n items.
R3. Yes. Finding the number of arrangements of n items with p of one type identical and q of another type identical is a valid solution. The result is the same $\frac{9!}{4!5!}$.

Practise

1. a) $_9C_0$ $_9C_1$ $_9C_2$ $_9C_3$ $_9C_4$ $_9C_5$ $_9C_6$ $_9C_7$ $_9C_8$ $_9C_9$
   b) $_4C_4$ $_5C_4$ $_6C_4$ $_7C_4$ $_8C_4$
2. $a = 286 + 78$ $= 364$ $b = 1001 - 286$ $= 715$ $c = a + 1001$ $= 364 + 1001$ $= 1365$
3. D
4. B
5.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

a) $_6C_3 = 20$ b) $_8C_4 = 70$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

c) ${}_4C_3 = 4$ **d)** ${}_7C_6 = 7$

Apply

6. a) i) 4 ii) 9 iii) 16
b) They are perfect squares.
c) These occur in diagonal 2.
d) Each perfect square greater than 1 is equal to the sum of a pair of adjacent terms on diagonal 2 of Pascal's triangle: $n^2 = {}_nC_2 + {}_{n+1}C_2$, $n > 1$

7. a) 165; ${}_7C_7 + {}_8C_7 + {}_9C_7 + {}_{10}C_7 = {}_{11}C_8$
b) ${}_rC_r + {}_{r+1}C_r + {}_{r+2}C_r + \ldots + {}_{r+k-1}C_r = {}_{r+k}C_{r+1}$

8. 35 **9.** 27 **10.** 180

11. a) 32; Since this is a triangular array, combinations can be used to solve this question.
b) 20; Since this is not a triangular array, combinations cannot be used.

12. a) diagonal 2
b) $1 + 2 = {}_3C_2$ $1 + 2 + 3 = {}_4C_2$ $1 + 2 + 3 + 4 = {}_5C_2$
The sum of the first n natural numbers is ${}_{n+1}C_2$.
c) Example 1 involved sums of terms in diagonal 2. This question involves sums of terms in diagonal 1.

13. a) $x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$
b) $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
c) $16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4$
d) $x^6 - 6x^4 + 12x^2 - 8$

14. a)

Number of Lines	Number of Regions	Rewrite the Number of Regions
0	1	1
1	2	1 + 1
2	4	1 + (1 + 2)
3	7	1 + (1 + 2 + 3)
4	11	1 + (1 + 2 + 3 + 4)
⋮		⋮
n		1 + (1 + 2 + 3 + ... + n)

The values being added represent the triangular numbers, whose sum is $\frac{n(n+1)}{2}$. The formula for the number of regions is $R(n) = 1 + \frac{n(n+1)}{2}$.
The sum of the first n natural numbers is ${}_{n+1}C_2$.
So, $R(n) = 1 + {}_{n+1}C_2$.
b) 211

15. a)

n	Sum of Squares $1^2 + 2^2 + \ldots + n^2$	$t_{n+1,3} + t_{n+2,3}$
1	$1^2 = 1$	
2	$1^2 + 2^2 = 5$	$t_{3,3} + t_{4,3} = 1 + 4 = 5$
3	$1^2 + 2^2 + 3^2 = 14$	$t_{4,3} + t_{5,3} = 4 + 10 = 14$
4	$1^2 + 2^2 + 3^2 + 4^2 = 30$	$t_{5,3} + t_{6,3} = 10 + 20 = 30$
5	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$	$t_{6,3} + t_{7,3} = 20 + 35 = 55$
6	$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$	$t_{7,3} + t_{8,3} = 35 + 56 = 91$

b) The values in columns two and three are the same.
c) The sum of the first n squares is ${}_{n+1}C_3 + {}_{n+2}C_3$, $n > 1$.
d) 42 925

16. a)

Layer, n	Total Number of Oranges	$t_{n+1,2} + t_{n+1,3}$
1	1	
2	1 + 3 = 4	$t_{3,2} + t_{3,3} = 3 + 1 = 4$
3	1 + 3 + 6 = 10	$t_{4,2} + t_{4,3} = 6 + 4 = 10$
4	1 + 3 + 6 + 10 = 20	$t_{5,2} + t_{5,3} = 10 + 10 = 20$

b) The total number of oranges needed for a stack of n layers can be found in diagonal 3 of Pascal's triangle.
c) ${}_{n+1}C_2 + {}_{n+1}C_3$, $n > 1$ **d)** 220

18. $(h + t)^5 = {}_5C_0h^5t^0 + {}_5C_1h^4t^1 + {}_5C_2h^3t^2 + {}_5C_3h^2t^3 + {}_5C_4h^1t^4 + {}_5C_5h^0t^5$
$= 1h^5t^0 + 5h^4t^1 + 10h^3t^2 + 10h^2t^3 + 5h^1t^4 + 1h^0t^5$
If a coin is tossed five times, there is
1 way to get 5 heads and 0 tails
5 ways to get 4 heads and 1 tail
10 ways to get 3 heads and 2 tails
10 ways to get 2 heads and 3 tails
5 ways to get 1 head and 4 tails
1 way to get 0 heads and 5 tails

Extend

19. a) $p^5 - 5p^3 + 10p - \frac{10}{p} + \frac{5}{p^3} - \frac{1}{p^5}$
b) $81m^8 + 216m^4 + 216 + \frac{96}{m^4} + \frac{16}{m^8}$

3.5 Probabilities Using Combinations, pages 128–133

Example 1 Your Turn
a) approximately a 0.000 113% chance
b) approximately a 0.015 765% chance
c) approximately 0.999 841 22
d) Answers may vary. It is extremely unlikely that anyone will win the lottery prizes.

Example 2 Your Turn
a) approximately 0.36 **b)** approximately 0.28
c) It is more likely that there will be equal numbers of male and female students than more female than male students.

Example 3 Your Turn
Slots A and F: 0; Slots B and E: $\frac{1}{8}$; Slots C and D: $\frac{3}{8}$

Reflect

R1. Answers may vary. A student selects three cards in order, without replacement, from a standard deck. What is the probability that the student selects a king, then two queens? What is the probability that a hand of three cards contains only face cards?

R2. If you interpret the language to mean Jake is first and Hamid is second, order matters. So, the probability that two are the top two finishers is $\frac{{}_2P_2}{{}_8P_2}$.
If you interpret the language to mean Jake and Hamid are top two with no assigned place (first or second), order does not matter. So, the probability that two are the top two finishers is $\frac{{}_2C_2}{{}_8C_2}$. Both expressions result in the same probability of $\frac{1}{28}$.

Practise

1. a) approximately 0.000 495
   b) approximately 0.025
   c) approximately a 0.000 305
2. $\frac{1}{3}$ 3. B 4. C

Apply

5. a) approximately 0.006 b) approximately 0.076
   c) approximately 0.002 d) approximately 0.7
6. approximately 1.575×10^{-12}
7. approximately 0.303
8. approximately 0.145 : 0.855
9. approximately 0.167
10. a) space D at about 0.38
    b) If the checker begins in a different location, the number of possible paths ending at each destination will be different.
11. approximately 0.908
12. Answers may vary. The probability of the disc landing in each slot at the bottom of the board depends on its starting slot. Dropping the ball from one of two centre slots (3 or 4) will give the most paths, so that is the best strategy. Starting in slot 3, the probability of the ball landing in A or F: $\frac{1}{32}$; B or E: $\frac{5}{32}$; C or D: $\frac{5}{16}$; G: 0.
13. approximately 0.54
14. approximately 0.476

Extend

16. $\frac{7}{9}$
17. a) approximately 0.952 b) approximately 0.952
    c) approximately 0.548
18. 0.225

Chapter 3 Review, pages 134–135

1. 840
2. a) 840 b) 3 326 400 c) 277 200
3. 1001
4. a) i) $r = 4$ ii) $r = 5$
      iii) $r = 3$ or $r = 4$ iv) $r = 7$ or $r = 8$
   b) The greatest number of combinations when n is even occurs at $r = \frac{n}{2}$. The greatest number of combinations when n is odd occurs at $r = \frac{n}{2} \pm 0.5$.
5. a) 352 800 b) 210
6. a) 210
   b) Answers may vary. A committee has 10 people. In how many ways could a president and vice president be chosen?
   c) Answers may vary. From a committee of 10 people, there are ${}_{10}P_2$, or 90 ways to choose a president and vice president.
7. 300
8. 31
9. a) 30 257 175 b) 22 120 065 c) 22 116 900
10. $a = 792$, $b = 462$
11. a) i) 1 ii) 5 iii) 15
    b) They are entries in diagonal 4 of Pascal's triangle.
    c) They are represented by ${}_nC_4$.
    d) 495
12. a) row 9 b) row 12
13. Pascal's Method:

Home	1	1	1	1
1	2	3	4	5
1	3	6	10	15
1	4	10	20	35
1	5	15	35	70
1	6	21	56	126
1	7	28	84	210
1	8	36	120	330

School

Stephen can take 330 different routes to school.
Combinations: ${}_{11}C_2 \times {}_7C_7 = 330$

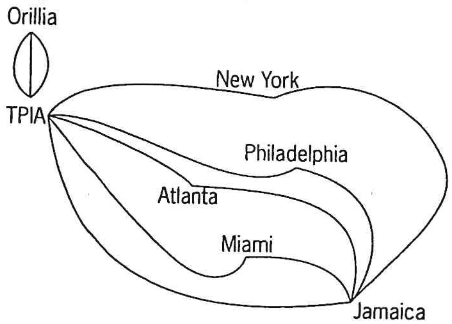
14. a) $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
    b) $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$
15. a) approximately 0.000 285
    b) approximately 0.000 495
16. a) approximately 0.005 b) approximately 0.587
17. a) approximately 0.004 b) approximately 0.496
    c) 0.504

Chapter 3 Test Yourself, pages 136–137

1. C 2. B 3. B 4. A
5. 1001 6. 35 7. 70 8. 15 120
9. a) ${}_8C_3 = \frac{{}_8P_3}{3!}$
   b) Both ${}_8C_3$ and ${}_8P_3$ represent the number of arrangements of 3 items from 8. However, combinations have no regard for order, while permutations do. Combination: A committee of three people can be chosen from a list of 8 people in ${}_8C_3$, or 56 ways. Permutation: From a committee of 8 people, there are ${}_8P_3$, or 336 ways to choose a president, vice president, and secretary.
10. approximately 0.396
11. 210
12. Permutations With Like Objects:
    $\frac{18!}{3!3!3!3!3!3!} = 137\ 225\ 088\ 000$
    Combinations:
    ${}_{18}C_3 \times {}_{15}C_3 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3 = 137\ 225\ 088\ 000$
13. a) Each row in Pascal's triangle represents the combinations of choosing 0 items, 1 item, 2 items, and so on, out of n items.
    b) The terms of Pascal's triangle are generated by adding two adjacent terms and placing the result immediately below them in the next row.
    $t_{n,r} + t_{n,r+1} = t_{n+1,r+1}$
    Using combinations, ${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$.
14. a) Alternately subtracting and adding successive terms in a row of Pascal's triangle results in 0.
    b) For $n > 0$, ${}_nC_0 - {}_nC_1 + {}_nC_2 - \dots {}_nC_n$.
15. a) 169
    b) It would be greater, since there are more chances to get olives or mushrooms. ${}_{15}C_4 - {}_{13}C_4 = 650 > 169$
16. a) approximately 0.81 b) approximately 0.008
    c) approximately 0.184
17. a) approximately 0.0002 b) approximately 0.043
    c) approximately 0.381 d) approximately 0.624
    e) approximately 0.351

Chapters 1 to 3 Cumulative Review, pages 138–139

1. a) $\frac{2}{3}$ b) $\frac{1}{3}$
2. a) 1:4 b) 1:1
3. a) Experimental probability is based on experimental trials, while theoretical probability is based on the analysis of all outcomes.
   b) Experimental probability approaches theoretical probability as a very large number of trials are conducted.
4. $\frac{2}{9}$
5. $\frac{7}{13}$
6. a) $\frac{9}{25}$ b) $\frac{3}{10}$
   c) The answers to parts a) and b) are different because one deals with replacement and the other does not.
7. Map:



Tree diagram outcomes: (O, D, NY, J), (O, D, M, J), (O, D, A, J), (O, D, P, J), (O, D, J), (O, B, NY, J), (O, B, M, J), (O, B, A, J), (O, B, P, J), (O, B, J), (O, T, NY, J), (O, T, M, J), (O, T, A, J), (O, T, P, J), (O, T, J)

List:

OT_cNJ	OT_cPJ	OT_cAJ	OT_cMJ	OT_cJ
OT_bNJ	OT_bPJ	OT_bAJ	OT_bMJ	OT_bJ
OT_tNJ	OT_tPJ	OT_tAJ	OT_tMJ	OT_tJ

8. a) 216 b) 1296 c) 64 d) 1728
9. a) 1680
   b) Adjacent countries share boundaries. These boundaries are more visible if the countries are different colours. With only 8 colours available, there could be many countries that are coloured the same colour, but adjacent countries should not be.
10. 124 251 000
11. 1680
12. 399 168 000
13. a) $\frac{1}{120}$
    b) Winning would be less probable if the digits could be repeated, because there would be more possible outcomes.
14. 63
15. 1 646 400
16. a) 210
    b) Answers may vary. ${}_{10}C_6 = 210$. Pascal's method will arrive at the same result by adding the number of paths to the adjacent grid points to determine the number of paths to the given point.
17. a)

n	${}_nC_2 \div {}_nC_1$	Result
2	1 ÷ 2	0.5
3	3 ÷ 3	1
4	6 ÷ 4	1.5
5	10 ÷ 5	2
6	15 ÷ 6	2.5
7	21 ÷ 7	3
8	28 ÷ 8	3.5
9	36 ÷ 9	4

    b) When n is odd, ${}_nC_2$ is divisible by ${}_nC_1$.
    c) When n is odd, ${}_nC_2$ is divisible by ${}_nC_1$. These rows have an even number of terms.
    d) Yes, ${}_{15}C_2$ is divisible by ${}_{15}C_1$, because n is odd.
18. 56
19. a) 31 b) 31
20. a) 6
    b) Yes. Let the three directions the spider can move be right, left, and down. The spider needs to travel 3 edges to its destination. Select any one of these three edges to travel, say right. From two remaining edges, select another direction, say left. Then, the last edge travel down.

$$ {}_3C_1 \times {}_2C_1 \times {}_1C_1 = \frac{3!}{2!1!} \times \frac{2!}{1!1!} \times \frac{1!}{0!1!} $$
$$ = 6 $$

21. a) approximately 0.190 b) approximately 0.9903

Chapter 4 Probability Distributions for Discrete Variables

Prerequisite Skills, pages 142–143

1. a) $\frac{4}{52}$ or $\frac{1}{13}$ b) $\frac{26}{52}$ or $\frac{1}{2}$ c) $\frac{13}{52}$ or $\frac{1}{4}$
2. a)

Sum	Possible Groupings	Number of Outcomes	Probability
3	(1,1,1)	1	$\frac{1}{216}$
4	(1,2,1)	3	$\frac{3}{216}$
5	(1,3,1), (1,2,2)	6	$\frac{6}{216}$
6	(1,4,1), (1,3,2), (2,2,2)	10	$\frac{10}{216}$
7	(1,4,2), (1,3,3), (5,1,1), (3,2,2)	15	$\frac{15}{216}$
8	(1,4,3), (1,2,5), (1,1,6), (4,2,2), (3,3,2)	21	$\frac{21}{216}$
9	(6,2,1), (5,3,1), (5,2,2), (4,4,1), (4,3,2), (3,3,3)	25	$\frac{25}{216}$
10	(6,3,1), (6,2,2), (5,3,2), (5,4,1), (4,4,2), (4,3,3)	27	$\frac{27}{216}$
11	(6,4,1), (6,3,2), (5,5,1), (5,4,2), (5,3,3), (4,4,3)	27	$\frac{27}{216}$
12	(6,5,1), (6,4,2), (6,3,3), (5,5,2), (5,4,3), (4,4,4)	25	$\frac{25}{216}$
13	(6,6,1), (6,5,2), (6,4,3), (5,5,3), (5,4,4)	21	$\frac{21}{216}$
14	(6,4,4), (6,5,3), (5,5,4), (6,6,2)	15	$\frac{15}{216}$
15	(6,6,3), (6,4,5), (5,5,5)	10	$\frac{10}{216}$
16	(6,6,4), (6,5,5)	6	$\frac{6}{216}$
17	(6,6,5)	3	$\frac{3}{216}$
18	(6,6,6)	1	$\frac{1}{216}$

   b) The sum of the probabilities is 1.

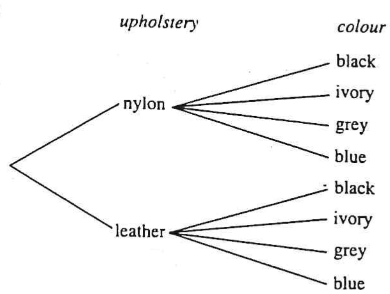
ANSWERS

CHAPTER 1 PERMUTATIONS

EXERCISE 1.1

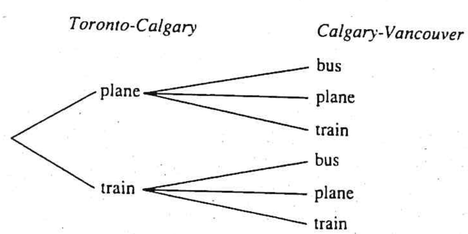
1. (a) Sum (b) Product (c) Product (d) Sum (e) Product

2. (a)



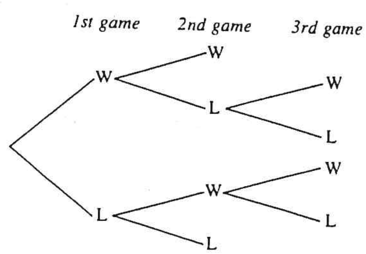
(b) 8

3. 6



4. 21 **5.** (a) 25 (b) 20 **6.** 60

7. (a)



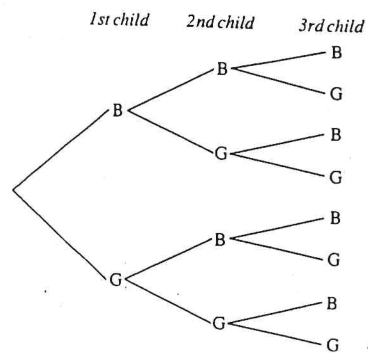
(b) 2

8. 47 **9.** 28 **10.** 1296 **11.** 24

12. 36 **13.** 208 860

14. (a) 4 (b) 13 (c) 16 (d) No **15.** 12

16. (a)



(b) 3 (c) 4 (d) No

EXERCISE 1.2

1. No

2. (a) – (v), (b) – (iii), (c) – (i), (d) – (vi), (e) – (iv), (f) – (ii)

3. (a) 336 (b) 19 535 040 (c) 5985 (d) 36 (e) 3 652 110 (f) 2 919 735

4. (a) 30 240 (b) 586 051 200 (c) 84 (d) 495 (e) 56 (f) 8008 (g) 20 (h) 990

6. (a) 720 (b) 6! **7.** 7!

8. (a) $n!$ (b) $(n+1)!$ (c) $(n+1)!$ (d) $(n+2)!$ (e) $n(n-1)$ or n^2-n (f) $(n+2)(n+1)n$

9. (a) 8 (b) 5 (c) 6 (d) 8

EXERCISE 1.3

1. (a) 60 (b) 15 120 (c) 1 663 200 (d) 120 (e) 1680 (f) 1 814 400

3. 24 **4.** (a) 60 (b) 125 (c) 12 (d) 50

5. 336 **6.** 40 320 **7.** 997 002 000

8. 1440 **9.** (a) 95 040 (b) 7920

10. (a) 6840 (b) 8000

12. (a) 40 320 (b) 5040 (c) 24 (d) 720

14. (a) (i) 20, 60 (ii) 336, 6720 (iii) 3 991 680, (b) 95 040 (c) (i) 1 663 200 (ii) 3024

15. (a) 20 (b) 10

EXERCISE 1.4

2. (a) 840 (b) 120 (c) 39 916 800 (d) 2 494 800 (e) 129 729 600 (f) 415 800

3. 44666 46466 46646 46664 64664 66464 66644 66446 64646 64466

4. (a) 210 (b) 35 **5.** 12 600 **6.** 84

7. 1287 **8.** 10 **9.** 369 600 **10.** 13 860

11. 45 045

12. (a) 453 600 (b) 45 360 (c) 90 720 (d) 90 720

13. 210 **14.** 39 916 800 × cost of stamp

15. (a) 7 484 400 (b) 369 600

EXERCISE 1.5

1. 39 916 800 **2.** 720 **3.** 30 240

4. (a) 4096 (b) 3584 (c) 625 (d) 4500 (e) 4500

5. 127 **6.** 14 400 **7.** 32 659 200 **8.** 60

9. 3439 **10.** 480 **11.** 239 500 800

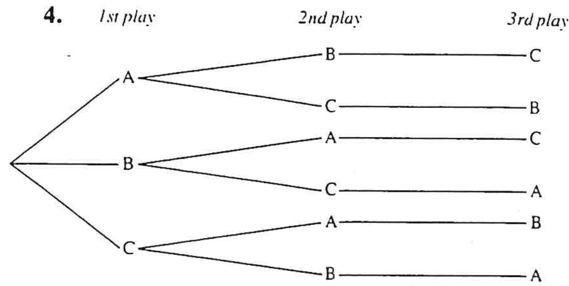
12. 40 319 **13.** 90 720 **14.** 144

15. 33 **16.** 26 **17.** 7200

1.6 REVIEW EXERCISE

3. (a) 5 (b) 1 (c) 1 (d) 6

4.



5. (a) 360 (b) 6720 (c) 72 (d) 1260

6. 676, 17 576 **7.** 5039

8. (a) 5040 (b) 45 360 (c) 25 200

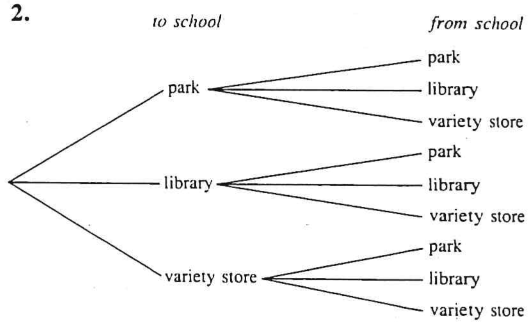
9. 2520 **10.** 5040

11. (a) 120 (b) 24 (c) 48 (d) 72 **13.** 6

1.7 CHAPTER 1 TEST

1. −46

2.



3. 210 **4.** 5040 **5.** (a) 42 (b) 252

6. (a) 420 (b) 120

7. 320 **8.** (a) 24 (b) 12 (c) 36

CHAPTER 2 COMBINATIONS

REVIEW AND PREVIEW TO CHAPTER 2

EXERCISE 1

1. (a) $\frac{m^2}{n^2}$ (b) $\frac{1}{2}$ (c) $\frac{y^2}{x^2}$ (d) $\frac{x+3}{x+4}$ (e) 1 (f) $\frac{x+y}{y}$

2. (a) $\frac{x(x-2)}{4(x+2)}$ or $\frac{x^2-2x}{4x+8}$ (b) $x(4x+1)$ or $4x^2+x$ (c) $\frac{1}{a^2+b^2}$

EXERCISE 2

1. (a) $\frac{x^2-x-7}{6}$ (b) $\frac{7x+3}{x-4}$

2. (a) 420 (b) x^2-1 (c) $8+4x-2x^2-x^3$

3. (a) $\frac{53a}{36}$ (b) $\frac{2y-3x-2x^2}{xy}$ (c) $\frac{19ab-2b^2+24a^2}{18ab}$ (d) $\frac{10x+2}{12+x-x^2}$ (e) $\frac{2a-ab+2b}{a-ab}$