Reflect

R1. Explain why you need to divide by 4! when calculating the number of arrangements of the digits 1, 2, 2, 2, 2, 3, 4.

R2. Is the number of permutations of three girls and four boys the same as the number of permutations of three red balls and four green balls? Explain.

R3. Why is it easier to use the formula $n(A) = \frac{n!}{p!\,q!\,r!\ldots}$ than to use a tree diagram or chart?

Practise

Choose the best answer for #2 and #3.

1. Simplify.

a) $\frac{10!}{2!\,3!\,5!}$

b) $\frac{9!}{3!\,3!\,3!}$

c) $\frac{7!}{2!\,3!}$

d) $\frac{120!}{115!\,3!\,2!}$

2. What is the number of arrangements of five small tiles and three large tiles?

A 20

B 56

C 720

D 40 320

3. Dana has 12 pens. There are four blue, three red, and the others are different colours. Which set of values for the variables in a permutation calculation is correct?

A $n = 12, p = 4, q = 3, r = 5$

B $n = 7, p = 4, q = 3$

C $n = 12, p = 4, q = 3, r = 0$

D $n = 12, p = 4, q = 3$

4. How many permutations are there of all the letters in each name?

a) WATERLOO

b) TORONTO

c) MISSISSAUGA

d) OTTAWA

5. How many five-digit numbers can be formed using each set of numbers?

a) 1, 2, 2, 3, 4

b) 1, 2, 2, 2, 3

c) 1, 1, 2, 3, 3

d) 1, 2, 2, 2, 2

Apply

6. Sam has four different types of fruit. He has three pieces of each type. In how many ways could he arrange them on a platter

a) in a line?

b) in three rows of four?

c) in two rows of six?

7. In a panel of eight light switches, half are on and half are off. In how many ways could this be done?

8. In one of her tricks, a clown rearranges two identical quarters, three identical loonies, and five identical toonies in a row. In how many ways can the clown arrange the coins?

9. How many arrangements are there of 15 flags in a row if five are red, four are green, two are blue, and four are yellow?

10. **Communication** When applying the formula $n(A) = \frac{n!}{p!\,q!\,r!\ldots}$, will the result ever not be a natural number? Justify your explanation. Hint: A natural number is a whole number greater than zero.

11. **Application** When travelling from home to school, Minh travels five blocks north and six blocks west. How many different routes can he take? What assumptions did you make?

12. **Thinking** In how many ways could 12 volleyball players be assigned to
    a) four triple rooms?
    b) six double rooms?

13. In how many ways could the letters in the word PROBLEM be arranged if the consonants must remain in the original order?

14. In how many ways could the digits in the number 458 978 be arranged if the prime digits must remain in the original position?

✓ Achievement Check

15. In the opening credits of each episode of the TV show *Fawlty Towers*, the sign on the front of the hotel rearranged the letters in the hotel's name.
    a) How many arrangements are there of all the letters in FAWLTY TOWERS?
    b) How many arrangements of these letters are possible if the A, Y, O, and E must remain in their original order?
    c) How many arrangements of these letters are possible if the vowels must remain in the second, sixth, eighth, and tenth positions?
    d) How many arrangements of these letters are possible if the consonants (excluding Y) must remain in alphabetical order?

16. **Thinking** An Ontario licence plate contains four letters followed by three digits. Derek noticed that the four letters on his licence plate were all different and in alphabetical order. Similarly, the three digits were all different and in numerical order.
    a) How many licence plates have all different letters and digits and just the letters in alphabetical order?
    b) How many licence plates have all different letters and digits and just the digits in numerical order?
    c) How many have all different letters and digits and both the letters in alphabetical order and the digits in numerical order?
    d) Mathematically, how does the number of licence plates in part c) compare to the total number of licence plates with no restrictions?

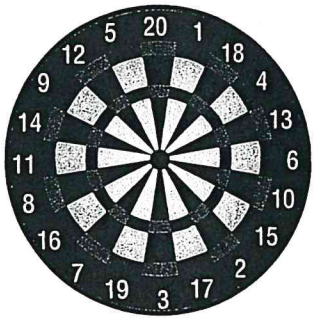
17. **Open Question** Design an example that has $\frac{12!}{2!\,3!\,4!\,3!}$ as its solution. Justify your reasoning.

Extend

18. How many 7-digit even numbers can be formed using all of the digits 0, 1, 1, 2, 3, 4, 5?

19. How many four-letter "words" can be made from the letters of the word APPLE?

20. A dart board has the numbers from 1 to 20 around the circumference. In how many ways could the numbers be arranged if pairs of numbers on opposite sides of the board must add to the same value?



EXERCISE 1.4

A **1.** Identify the indistinguishable items in each situation. (Class discussion of various interpretations of the given information would be useful.)

(a) The letters of the word LETTUCE are rearranged.

(b) Several copies of textbooks for the Grade 9 and Grade 10 Math courses are lined up on a shelf.

(c) At a buffet table, there are four pieces of cheesecake, two dishes of chocolate mousse, and three chocolate eclairs.

(d) The digits 4, 5, 6 are used, twice each, to create six-digit numbers.

B. **2.** In how many ways can the letters of each word be arranged?

(a) MAXIMUM (b) CANADA

(c) SASKATCHEWAN (d) INTERESTING

(e) UNINTERESTING (f) MISSISSAUGA

3. List all five-digit numbers that can be formed by using two 4's and three 6's.

4. How many seven-digit integers are there which include

(a) two 3's, three 2's, and two 8's?

(b) four 3's and three 4's?

5. A man bought two vanilla ice cream cones, three chocolate cones, four strawberry cones, and one pistachio cone for his ten children. In how many ways can he distribute the flavours among the children?

6. A coin is tossed nine times. In how many ways could the results be six heads and three tails?

7. Anya is starting out on her evening run. Her route always takes her eight blocks east and five blocks north to her grandmother's apartment building. But she likes to vary the path she follows. How many different possibilities does she have?
[Hint: Consider the permutations of 13 letters, 8E's and 5 N's.]

8. How many numbers greater than 300 000 are there using only the digits 1, 1, 1, 2, 2, 3?

9. Yurak is shelving books in a display in the school library. He has four different books with three copies of each. In how many ways can he arrange the books on the shelf for display?

10. A developer will build 12 houses on the same side of Costly Court in a new subdivision. If he has room for two houses modeled on Plan A, four modeled on Plan B, and six modeled on Plan C, in how many different ways can he arrange the houses on the street?

11. Emily's minor soccer team played a total of 14 games in the season. Their record was eight wins, four losses, and two ties. In how many orders could this have happened?

12. (a) How many permutations are there of the letters of the word BASKETBALL?
(b) How many of the arrangements begin with K?
(c) How many of the arrangements start with a B?
(d) In how many of the arrangements would the two L's be together?

C **13.** In how many ways can the letters of the word SECTION be arranged if the consonants must always be in the order in which they occur in the word itself?

14. A soap company will give away a million dollars to anyone who guesses the order into which they have rearranged the letters of the word SUPERCLEANER. Each entry must be in a separate envelope. How much would it cost you in postage to submit all possible entries?

15. In how many ways can 12 basketball players be assigned to
(a) six double rooms? (b) four triple rooms?