

Consolidate and Debrief

Key Concepts

- The number of permutations of n items is n factorial,

$$n! = n(n-1)(n-2)(n-3) \dots \times 3 \times 2 \times 1$$

- You can use factorials as a counting technique when repetition is not permitted.
- The number of r -permutations of n items can be calculated by

$$\begin{aligned} {}_n P_r &= n(n-1)(n-2) \dots (n-r+1) \\ &= \frac{n!}{(n-r)!} \end{aligned}$$

Reflect

- R1.** Which would have more possibilities, arranging r people from a group of n people **with** regard to order or **without** regard to order? Explain your reasoning.
- R2.** Use your calculator to determine the value of $0!$. Explain why it would have this value. Include an example to support your explanation.

Practise

Choose the best answer for #4 and #5.

1. Evaluate.

- a) $9!$
- b) $\frac{12!}{5!}$
- c) ${}_7 P_7$
- d) ${}_8 P_5$

2. Write in factorial form.

- a) ${}_6 P_4$
- b) ${}_{15} P_6$
- c) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- d) $8 \times 7 \times 6 \times 5$
- e) $n(n-1)(n-2)(n-3)$
- f) $(n+1) \times (n) \times (n-1) \times \dots \times 3 \times 2 \times 1$

3. Express in the form ${}_n P_r$.

- a) $6!$
- b) $91 \times 90 \times 89 \times 88 \times 87 \times 86$
- c) $\frac{18!}{12!}$

4. Which is the correct simplification of $\frac{96!}{24!}$?

- A $4!$
- B 4
- C ${}_{96} P_{72}$
- D ${}_{96} P_{24}$

5. Which is the correct number of permutations of five items from a list of nine items?

- A 126
- B 15 120
- C 45
- D 59 049

6. There are 15 teams competing in a synchronized swimming competition. In how many ways could first, second, and third place be awarded?

7. A club has 18 members. In how many ways could a president, vice president, treasurer, and secretary be elected?

Apply

8. There are 22 players on a baseball team. In how many ways could the batting order of nine players be assigned?
9. Write in simplest factorial form.
- $10 \times 9 \times 8 \times 7!$
 - $99 \times 98 \times 97!$
 - $90 \times 8!$
 - $n(n-1)!$
 - $(n+2)(n+1)n!$
10. **Application** A salesperson needs to visit 15 different offices during the week.
- In how many ways could this be done?
 - In how many ways could she visit four different offices on Monday?
 - In how many ways could she visit three different offices each day from Monday to Friday?
11. a) How many 10-digit numbers are there with no digits repeated?
b) How many 7-digit numbers are there with no digits repeated?
12. Caleb needs to create an 8-digit password using only numbers. How many different passwords are there if he wants to use 00 exactly once?

✓ Achievement Check

13. The six members of the student council executive are lined up for a yearbook photo.
- In how many ways could the executive line up?
 - In how many ways could this be done if the president and vice president must sit together?
 - In how many ways could this be done if the president and vice president must sit together in the middle of the group?
14. How many ways are there to seat six boys and seven girls in a row of chairs so that none of the girls sit together?
15. **Thinking** Twenty figure skaters are in a competition. In the final round, the bottom five competitors skate first in a random order. The next five do likewise, and so on until the top five skate last in a random order. In how many ways could the skating order be assigned?

Extend

16. Solve for n .
- ${}_n P_2 = 110$
 - $P(n, 3) = 5!$
17. Ten couples are being seated in a circle. How many different seating arrangements are there if each couple must sit together?
18. The names of the Knights of the Round Table at Winchester, UK, were engraved on the table, but they are no longer visible. There are 23 knights, plus King Arthur himself. In how many ways could King Arthur and the knights be seated at the Round Table?



19. A double factorial represents the product of all odd, or even, integers up to a given odd number, n . For example,
 $9!! = 1 \times 3 \times 5 \times 7 \times 9$.
- Express $9!!$ as a quotient of factorials.
 - Express $(2k+1)!!$ as a quotient of factorials.
 - Simplify $(2n)!!$, writing it in simple factorial form.
20. Without using a calculator, determine how many zeros occur at the end of $30!$.