Key Concepts

- The total number of subsets of a set of n elements is $_nC_0 + {_n}C_1 + \ldots + {_n}C_n = 2^n$.

- In some cases the null set is not considered. In such cases, $_nC_1 + {_n}C_2 + \ldots + {_n}C_n = 2^n - 1$.

- Consider using the indirect method, especially if it involves fewer cases, such as when you need to choose at least one or two items.

- If the order is important, consider selecting the items first and then arranging them in order.

Reflect

R1. When determining the total number of subsets of a set, you add the number of possibilities in each case. Explain why you add instead of multiply.

R2. When using cases to determine the number of ways of selecting objects from different sets, do you multiply or add? Explain your reasoning.

R3. You can solve counting problems using powers, permutations, combinations, or both. Make a summary and a flowchart to help decide which method(s) to use. Include simple examples to support your summary.

Practise

Choose the best answer for #2 and #3.

1. How many different sums of money can be made from a $5 bill, a $10 bill, a $20 bill, and a $50 bill?

2. In how many ways could a group of 10 people form a committee with at least 8 people on it?

   A 45 **B** 56 **C** 450 **D** 1016

3. If a set has 12 elements, how many subsets can be formed?

   A 12 **B** 24 **C** 4095 **D** 4096

4. A judging panel will have 6 members chosen from 8 teachers and 10 students. There must be at least 3 students on the panel. In how many ways could there be

   a) 3 students on the panel?

   b) 4 students on the panel?

   c) 5 students on the panel?

   d) least 3 students on the panel?

Apply

5. **Communication** Identify whether the following situations involve permutations, combinations, or both. Justify your choice.

   a) forming a committee of 5 people from a group of 12 people

   b) choosing a president, a vice president, and a treasurer from a committee of 12 members

   c) choosing 4 men and 4 women to be on a basketball team from among 6 men and 6 women, and assembling the athletes for a team photo

   d) naming 3 people from among 15 contestants to win 3 different prizes

6. You receive requests to connect with people every day on your social media account. If you have 15 requests to be "friends" with people, in how many ways could you respond by either accepting or rejecting each request?

7. Tonya has the following toppings available for her sandwich: lettuce, tomatoes, onions, olives, sprouts, peppers, mustard, and shredded cheese. She can use up to three toppings. How many different sandwiches can Tonya make?

8. Rohan is shopping for new pants. Six different styles are available. How many different purchases could Rohan make?

9. You can factor the number 210 into prime factors as $2 \times 3 \times 5 \times 7$. The products of prime factors form divisors (e.g., $2 \times 3 = 6$). Determine the total number of divisors of 210.

10. A board of directors needs to assign a chair, vice chair, treasurer, secretary and communications officer. There are four women and six men on the board. There will be two women and three men on the executive. In how many ways could this be done?

11. **Thinking** In cribbage, each player is dealt six cards from a standard deck. In how many ways could a hand contain

    a) at least two queens?

    b) more than three red cards?

    c) at least two hearts and at least two spades?

12. **Application** A telemarketer will call 12 people from a list of 20 men and 25 women. In how many ways could he select

    a) 12 men or 12 women?

    b) 6 men and 6 women?

13. A cabin has two rooms with three single beds each, one room with four single beds, and one room with two single beds. Six girls and six boys are assigned to rooms with people of the same gender. In how many ways can the rooms be assigned?

14. Six students from each of grades 9 to 12 have been pre-selected to win eight different prizes as students of the month. In how many ways could two students from each grade be selected to win these prizes?

15. **Thinking** Given the numbers –6, –5, –4, –3, –2, –1, 1, 2, 3, 4, 5, in how many ways could four different numbers be chosen so that their product is negative?

✓ Achievement Check

16. On a crown and anchor wheel, a crown, an anchor, and the four suits from a deck of cards are displayed in slots around the wheel.
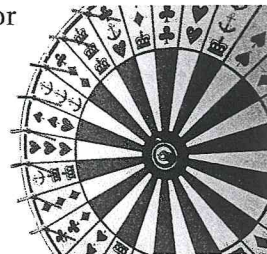
    a) Each three-of-a-kind (e.g., ♚♚♚) occurs twice. Calculate the number of slots with three-of-a-kind.

    b) Determine the number of slots with two-of-a-kind.

    c) The following restrictions are in place when all three symbols are different:
    - A crown and an anchor do not occur together (e.g., ♚⚓♦ cannot occur).
    - Three different suits do not occur together (e.g., ♦♥♠ cannot occur).
    - If a crown occurs with two different suits, an anchor may not also occur with the same two suits, and vice versa (only one of ♠⚓♦ or ♠♚♦ can occur).

    Calculate the number of slots with three different symbols. Use your calculations to verify the total number of slots on the wheel.

Extend

17. There are 10 points in a plane. No three points are collinear. How many convex polygons can be drawn using these points as vertices?

18. Five men and five women are selected from eight men and nine women and then seated around a circular table. In how many ways can this be done if their particular seat at the table does not matter?