Consolidate and Debrief

Key Concepts

- The rule of sum states that if one mutually exclusive event can occur in m ways, and a second can occur in n ways, then one **or** the other can occur in $m + n$ ways.
- If two events are not mutually exclusive, the principle of inclusion and exclusion needs to be considered:

  $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.
- To reduce calculations, consider using the indirect method, which involves subtracting the unwanted event from the total number of outcomes in the sample space: $n(A) = n(S) - n(A')$.

Reflect

R1. A student council has five executive positions: president, vice president, secretary, treasurer, and assistant treasurer. There are six female and seven male candidates. It is important that at least one male and one female be on the executive. Explain why the indirect method is useful in determining the number of possible outcomes.

R2. Write a general guideline explaining when to use the fundamental counting principle and when to use the rule of sum for permutations. Include a similar example for each.

Practise

Choose the best answer for #3 and #4.

1. Determine the total number of arrangements of three or four toys from a basket of eight different toys.

2. a) How many ways are there to roll a sum of 7 or 11 on two dice?

b) How many ways are there to roll doubles or a sum divisible by three on two dice?

3. A game has players roll either one or two standard dice. Which is the total number of possible different outcomes?

A 42

B 36

C 18

D 12

4. Which is the total number of arrangements of the digits 1, 2, 3, 4, 5, if the even digits must not be together?

A 120

B 24

C 48

D 72

Apply

5. a) How many even numbers can be formed from the digits 1, 2, 3, 4, 5?

b) How many of these numbers are greater than 3000?

6. Application A motorcycle licence plate consists of two or three letters followed by four digits. How many licence plates can be made?

7. A security code consists of either five or six different letters. How many distinct security codes are possible?

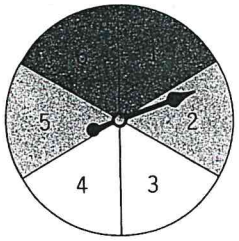
8. **Communication** Suppose a country has a rule that a newborn child may have either one, two, or three names.
   a) If parents were to choose from a list of 50 names, how many choices would they have when naming their child?
   b) What if they could choose from 100 names?
   c) Explain why the total in part b) is more than 2 times the total in part a).

9. **Open Question** Five speakers, P, Q, R, S, and T, have been booked to address a meeting.
   a) In how many ways could the speakers be ordered if speaker P must go before speaker Q?
   b) Make up your own problem about these five speakers. Solve it, share it with a classmate, and check his or her solution.

10. How many five-digit numbers include the digits 4 or 6 or both?

11. Ten names are placed into a hat. In how many ways could they be pulled from the hat so they are not in alphabetical order?

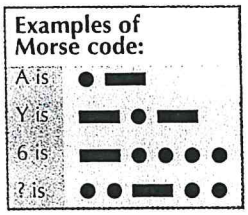
12. A spinner has six equally spaced sections numbered 1 to 6 as shown. You spin the spinner four times.



   a) In how many ways could the spinner result in the same colour on all four spins?
   b) In how many ways could the spinner result in an even number or the same colour on all four spins?

13. **Thinking** In the game of Monopoly, you can get out of jail by rolling doubles. If you are unsuccessful on the first roll you may try again, up to a total of three attempts. In how many ways could this occur? Explain your solution.

14. **Communication** Morse code uses dots and dashes to represent letters, digits, and eight punctuation symbols. Use the fundamental counting principle and the rule of sum to help explain why a maximum of six characters is needed.



✓ Achievement Check

15. A password must be 6, 7, or 8 characters long, and may include capital letters, lowercase letters, or digits. In how many ways could this be done
   a) with no restriction?
   b) with no repetition permitted?
   c) if at least one of the characters in part a) must be a digit?

Extend

16. How many different numbers can be formed by multiplying some or all of the numbers 2, 3, 4, 5, 6, 7, 8?

17. A derangement is a permutation of a set of numbers in which no item remains in its original position. For the set {1, 2, 3}, the derangements are {2, 3, 1} and {3, 1, 2}. The permutation {1, 3, 2} is not a derangement because 1 is in its original position. Determine the number of derangements of each set.
   a) {1, 2, 3, 4}
   b) {1, 2, 3, 4, 5}

18. The labels from six different cans of soup have come off. If you were to replace them at random, in how many ways could this be done so that
   a) none of the cans will be labelled correctly?
   b) at least one of the cans will be labelled correctly?
   c) all of the cans will be labelled correctly?