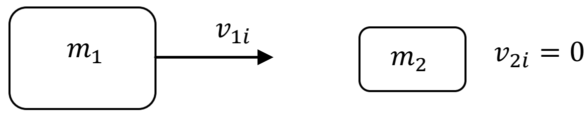
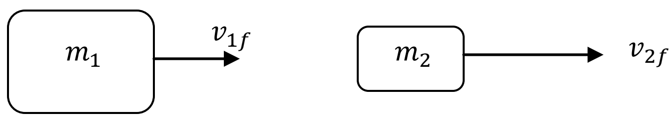
Solving Elastic Collision Problems by Change of Reference Frame

Consider the special case of completely elastic collision between two objects where the initial speed of the first object is zero.

Before



After



We have learned that during a completely elastic collision between two objects that both momentum and kinetic energy are conserved. Writing the equations for these conservations mathematically for the situation described above yields:

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Subbing in $v_{2i} = 0$,

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad ①$$

Kinetic energy conservation yields,

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Subbing in $v_{2i} = 0$ and multiplying everything by 2,

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \quad ②$$

Rearranging equation ② yields,

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 v_{2f}^2 \quad ③$$

Rearranging ① yields

$$m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \quad ④$$

Dividing equation ③ by equation ④

$$\frac{m_1(v_{1i}^2 - v_{1f}^2)}{m_1(v_{1i} - v_{1f})} = \frac{m_2 v_{2f}^2}{m_2 v_{2f}}$$

It is left up to the student to show that after completing the square and simplifying, one can sub in the result for v_{2f} into equation ① to yield the result

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$$

Also, from the result above, rearrange the equation for v_{1f} and sub it into equation ① to yield.

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

It is interesting from this point to examine three interesting cases resulting from these equations:

Case 1:
When the moving object has the greater mass ($m_1 > m_2$)

When the moving object has a much greater mass ($m_1 \gg m_2$)

Case 2:
When the moving object has a smaller mass ($m_1 < m_2$)

When the moving object has a much smaller mass ($m_1 \ll m_2$)

Case 3:
When the moving object has the same mass ($m_1 = m_2$)

Demonstration

Your teacher will perform a number of interesting demonstrations for each of the specific cases give above.

Examples

Basic Problem

1. An air track glider of mass 0.2 kg, moving at 1.0 m/s, collides elastically with another glider of mass 0.05 kg, which is initially at rest. What are the velocities of each glider after the collision?

   - Solve using the equations derived above (exactly as given)
   - Write the momentum and kinetic energy equations and solve graphically using WolframAlpha (www.wolframalpha.com)

Fixed Frame vs. Moving Frame of Reference

2. An air track glider of mass 3 kg, moving at 2.0 m/s [right], collides elastically with another glider of mass 1 kg, which is moving at 0.2 m/s [left]. What are the velocities of each glider after the collision?

   - Solve using the equations derived above – you will have to change the reference frame from a fixed frame to a frame moving with one of the objects.
   - Write the momentum and kinetic energy equations and solve graphically using WolframAlpha (www.wolframalpha.com)

WolframAlpha Solutions for Example 2

WolframAlpha computational knowledge engine

3(2)+1(-.2)=3a+1b and 3(2)^2+1(-0.2)^2=3a^2+1b^2

Input the momentum and kinetic energy equations that represent the system.

Input:

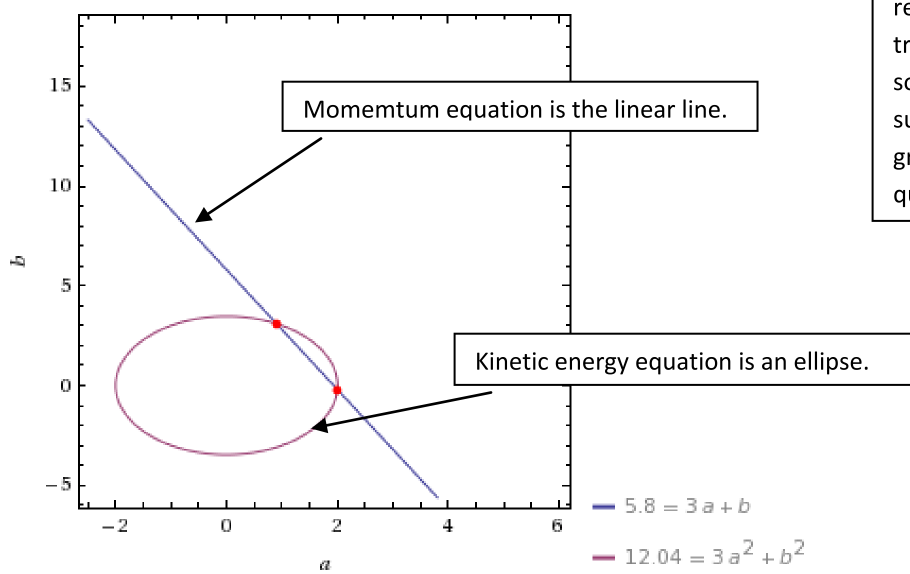
$$3\times 2+1\,(-0.2)=3\,a+1\,b\,\bigwedge\,3\times 2^2+1\,(-0.2)^2=3\,a^2+1\,b^2$$

Result:

$$5.8=3\,a+b\,\bigwedge\,12.04=3\,a^2+b^2$$

Wolfram interprets this as finding the intersection of the two equations – it shows what you input and the numerical result. The solution is traditionally discovered by solving for a variable and using substitution. The alternate graphical result is very clear and quite meaningful.

Plot of solution set:



Alternate form:

$$3\,a+b=5.8\,\bigwedge\,3\,a^2+b^2=12.04$$

Solutions:

$a\approx 0.9,\quad b\approx 3.1$

$a\approx 2.,\quad b\approx -0.2$

The two solutions (points of intersection) give the **final state** of the system after collision and the **initial state** of the system before collision and.

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