Solving Linear Systems - Cramer's Rule

Name: _____ Date: _____

Solving linear systems is something you have been doing since grade 9. You have used the substitution and elimination methods in the past. You can also use matrices to solve linear systems and actually they are very useful for solving higher order linear systems where there could be 4 or more unknowns with 4 equations. We will start with 2x2 systems and work to 3x3 systems.

Cramer's Rule for a 2x2 System

You can use determinants to solve a system of linear equations. This method uses the **coefficient matrix** of the linear system.

Linear System

$$\boldsymbol{a}x + \boldsymbol{b}y = e$$
$$\boldsymbol{c}x + \boldsymbol{d}y = f$$

Coefficient Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Let *A* be the coefficient matrix of this linear system:

$$ax + by = e$$
$$cx + dy = f$$

If $detA \neq 0$, then the system has exactly one solution. The solution is:

$$\boldsymbol{x} = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{detA} \quad \text{and} \quad \boldsymbol{y} = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{detA}$$

Cramer's Rule for a 3x3 System

Let *A* be the coefficient matrix of this linear system:

$$ax + by + cz = j$$
$$dx + ey + fz = k$$
$$gx + hy + iz = l$$

If $detA \neq 0$, then the system has exactly one solution. The solution is:

$$\boldsymbol{x} = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{detA} \quad \text{and} \quad \boldsymbol{y} = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{detA} \quad \text{and} \quad \boldsymbol{z} = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{detA}$$

Solve this using Substitution

Recall that solving the system means finding the point of intersection (where the two lines meet). Note that if the lines are parallel they will not meet, or if they are the same line you will get an infinite number of solutions instead of just a point.

$8x + 5y = 2$ and $2x - 4y = -10$

Solve Using Cramer's Rule (2x2 System)

$8x + 5y = 2$ and $2x - 4y = -10$

Solve Using Cramer's Rule (3x3 System)

$x + 2y - 3z = -2$
$x - y + z = -1$
$3x + 4y - 4z = 4$